



DEPARTMENT OF EDUCATION

GRADE 12 MATHEMATICS B

12.1: MEASUREMENT



FODE DISTANCE LEARNING



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FOR THE DEPARTMENT OF EDUCATION
PAPUA NEW GUINEA



GRADE 12

MATHEMATICS B

UNIT MODULE 1

MEASUREMENT

TOPIC 1: SCALES AND DIMENSIONS

TOPIC 2: PERIMETER AND AREA OF TRIANGLES

TOPIC 3: SURFACE AREAS AND VOLUME

TOPIC 4: SURVEYING



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MR. DEMAS TONGOGO
Principal-FODE



Flexible Open and Distance Education
Papua New Guinea

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SECRETARY'S MESSAGE

Achieving a better future by individuals students, their families, communities or the nation as a whole, depends on the curriculum and the way it is delivered.

This course is part and parcel of the new reformed curriculum – the Outcome Base Education (OBE). Its learning outcomes are student centred and written in terms that allow them to be demonstrated, assessed and measured.

It maintains the rationale, goals, aims and principles of the National OBE Curriculum and identifies the knowledge, skills, attitudes and values that students should achieve.

This is a provision of Flexible, Open and Distance Education as an alternative pathway of formal education.

The Course promotes Papua New Guinea values and beliefs which are found in our constitution, Government policies and reports. It is developed in line with the National Education Plan (2005 – 2014) and addresses an increase in the number of school leavers which has been coupled with a limited access to secondary and higher educational institutions.

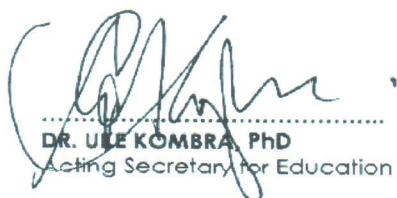
Flexible, Open and Distance Education is guided by the Department of Education's Mission which is fivefold;

- to facilitate and promote integral development of every individual
- to develop and encourage an education system which satisfies the requirements of Papua New Guinea and its people
- to establish, preserve, and improve standards of education throughout Papua New Guinea
- to make the benefits of such education available as widely as possible to all of the people
- to make education accessible to the physically, mentally and socially handicapped as well as to those who are educationally disadvantaged

The College is enhanced to provide alternative and comparable path ways for students and adults to complete their education, through one system, many pathways and same learning outcomes.

It is our vision that Papua New Guineans harness all appropriate and affordable technologies to pursue this program.

I commend all those teachers, curriculum writers and instructional designers, who have contributed so much in developing this course.



DR. ULE KOMBRA, PhD
Acting Secretary for Education



UNIT 1: MEASUREMENTS

INTRODUCTION

Measurement is the assignment of a numerical value to an attribute of an object, such as the length of a pencil, the height of a boy, or the weight of a book.

The study of measurement is important in the mathematics curriculum because of the practicality and wide use of measurement in many aspects of everyday life. The study of measurement also offers an opportunity for learning and applying other mathematics, including number operations, geometric ideas, statistical concepts, and notions of function. It highlights connections within and between mathematics and areas outside of mathematics, such as social studies, science, art, and physical education.

Measurement is best applied with the use of concrete materials. Students may not gain a good understanding of measurement without handling materials, making comparisons physically, and measuring with tools. Measurement concepts should grow in sophistication and breadth across the grades, and instructional programs should not repeat the same measurement curriculum year after year.

Measurement: Definition and Meaning

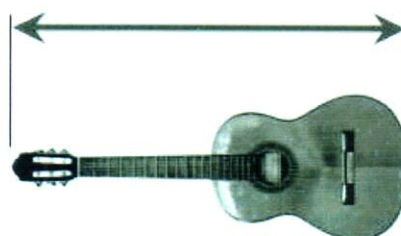
Measurement is the procedure or method of identifying the relationship of two quantities. It is the method of finding the ratio of substances that can be expressed in terms of numbers such as area, length, volume, mass, time, etc.

Measurement is finding a number that shows the size, quantity or amount of something. We can measure...

Length

Length is how far from end to end. It is also called distance.

The length of this guitar is about 1 metre.



Area

Area is the size of a surface: how much is inside the boundary of a flat (2-dimensional) object such as a triangle or circle.

Here Brownie, the dog is inside 1 square metre.





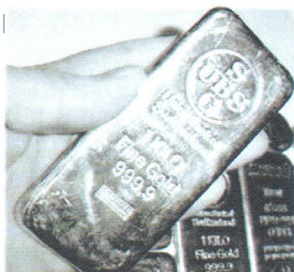
Volume

Volume is the amount of 3-dimensional space an object occupies. It is also called capacity.

This jug has exactly 1 litre of water.



Mass



Mass is how much matter something contains.

This bar of gold has a mass of 1 kilogram . (Weight is heaviness or the downward force caused by gravity on an object.)

Temperature

Temperature is how hot or cold an object or substance is. Temperature is measured using a thermometer, usually in the Celsius or Fahrenheit scale.

This block of ice would measure 0° **Celsius**, or 32° **Fahrenheit**



Time

Time is the ongoing sequence of events taking place. We measure time using seconds, minutes, hours, days, weeks, months, years, decades, centuries and millennium. There are many other methods to measure objects, but those discussed earlier are the most common.

History of Measurement

Years ago people came up with standard measure of length but they did not all agree on one system.

The Imperial System (which uses yards, feet, inches, etc. to measure length) was developed hundreds of years ago in the UK. Then the French came up with the **Metric System** (origins in 1670, but developed in the 1790s) which soon spread through Europe, and then most of the world, including England itself in 1965. The USA developed its own version of the **Imperial system** (US Standard Units), but the Metric System is also used in the USA, particularly in Science.

In most countries including Papua New Guinea, the system of measurement used is the Metric System.



LEARNING OUTCOMES

On successful completion of this module, you will be able to:

- identify the relationships between the corresponding sides in two similar figures and their areas
- calculate perimeter and areas of similar figures
- calculate actual areas of a field given the ratios
- calculate area of triangle using Pythagorean Theorem
- calculate the area of triangle using the sine ratio and the Heron's formula
- calculate perimeter and area of polygons by identifying triangular shapes
- calculate surface areas and volumes of pyramids, spheres and truncated solids
- determine pace length using a tape measure or metre ruler and trundle wheel
- estimate distances using pace length
- define survey lines, offset and field book
- interpret and calculate areas and perimeter using sketch or scale diagram
- describe offset method and triangulation and apply these methods to survey around obstacles



Time Frame

This unit should be completed within 10 weeks.

If you set an average of 3 hours per day, you should be able to complete the unit comfortably by the end of the assigned week.

Try to do all the learning activities and compare your answers with the ones provided at the end of the unit. If you do not get a particular exercise right in the first attempt, you should not get discouraged but instead, go back and attempt it again. If you still do not get it right after several attempts then you should seek help from your friend or even your tutor. Do not submit any question without solving it first.



12.1.1 SCALES AND DIMENSIONS

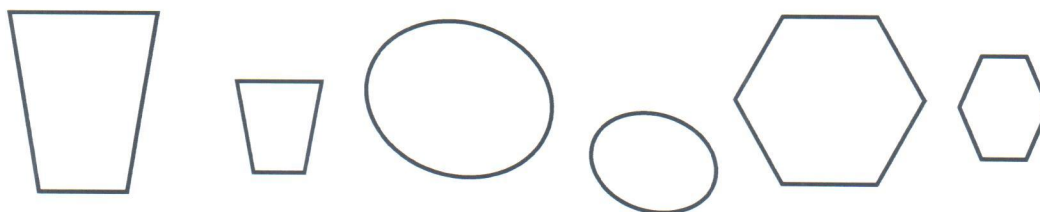
Have you ever read a road map or looked at plans for a house? The map or the plan is a scaled down version of the road or house. When two objects are identical, except one is a reduction or an enlargement of the other, the objects are said to be similar. Maps and plans are examples of similarity.

Maps and plans both use a scale. The scale tells us how many times larger an object is in reality compared to a plan. A **scale factor** is a number which scales or multiplies some quantity. We shall study scale factors having to do with **perimeters** and **areas**, and many others.

12.1.1.1: Areas and Scales

Similar Figures

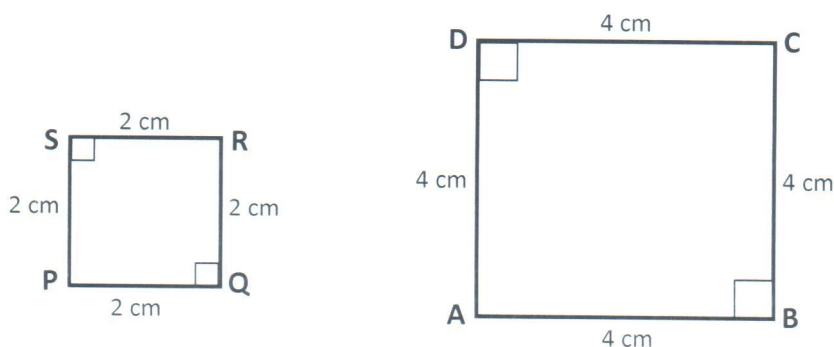
Two figures or shapes may be identical in shape but generally not in size. The following pairs of shapes will always be similar in shape but not in size.



Similar figures have the same shape (but not necessarily the same size) and the following properties:

- Corresponding sides are proportional, that is, the ratios of the corresponding sides are equal.
- Corresponding angles are equal.

For example, consider the following squares:





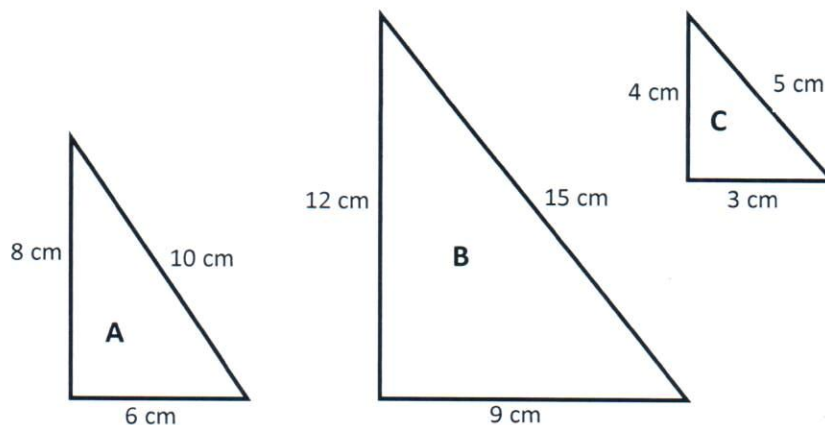
Solution:

Clearly, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CD}{RS} = \frac{DA}{SP} = 2$ (These are the corresponding sides that are proportional.)

Also, $\angle A = \angle P$; $\angle B = \angle Q$; $\angle C = \angle R$; $\angle D = \angle S$ (These are the corresponding angles that are equal.)

Thus the squares are similar figures as their corresponding sides are proportional and their corresponding angles are equal.

Similar shapes are those which are **enlargements** of each other. For example, the three triangles shown below are similar.



For example, to get triangle A from triangle C, we **enlarge C by multiplying** all the sides by a scale factor of 2. Triangle A is an enlargement of C.

Similarly to get triangle C from triangle A, we **reduce A by multiplying** all the sides by a scale factor of $\frac{1}{2}$. Triangle C is a reduction of A. Thus A and C are similar triangles.

A shape can be transformed into another similar shape by adjusting its size through enlargement or reduction by using a scale factor. In this process all dimensions of the shape must stay proportional to each other. (Example: the lengths, heights, or widths).

Enlargement

In mathematics, an **enlargement** is a uniform scaling (transformation) that **increases** distances, areas and volumes.

For example, a scale factor of 2 means that the new shape is twice the size of the original shape. A scale factor of 3 means that the new shape is three times the size of the original shape.



Reduction

Reduction is the opposite of enlargement where distances, areas and volumes are **decreased** using a uniform scaling.

To carry out enlargements or reductions you must first find the scale factor.

To find the enlargement scale factor, you divide the dimensions of the big shape by the small shape.

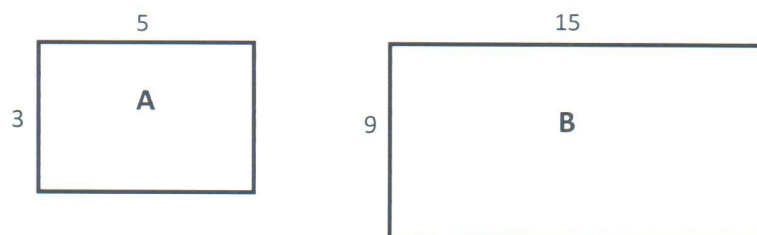
$$SF_{\text{enlargement}} = \frac{\text{BIG}}{\text{SMALL}}$$

To find the shape reduction scale factor, you divide the small dimensions by the big dimensions.

$$SF_{\text{reduction}} = \frac{\text{SMALL}}{\text{BIG}}$$

Example 1

What is the scale factor of the two figures below from A to B and from B to A?



Solution:

The scale factor of figure A to B is an enlargement scale factor.

$$SF_{\text{enlargement}} = \frac{\text{BIG}}{\text{SMALL}} = \frac{9}{3} = 3 ; \quad \frac{15}{5} = 3. \text{ Therefore the scale factor is } 3.$$

This means that figure A is **enlarged three times** to have figure B.

The scale factor of figure B to A is a reduction scale factor.

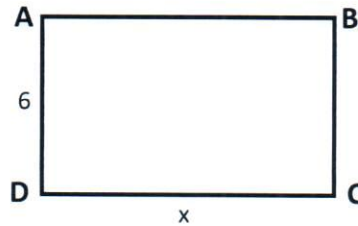
$$SF_{\text{reduction}} = \frac{\text{SMALL}}{\text{BIG}} = \frac{3}{9} = \frac{1}{3} ; \quad \frac{5}{15} = \frac{1}{3} \quad \text{The scale factor of B to A is } \frac{1}{3}.$$

This means that figure B is **reduced** by one third to have figure A.



Example 2

The following shapes **abcd** and **ABCD** are similar. Find the length of **CD**.



Solution:

Here we have to find the length of the bigger rectangle. We shall therefore be calculating an enlargement scale factor first.

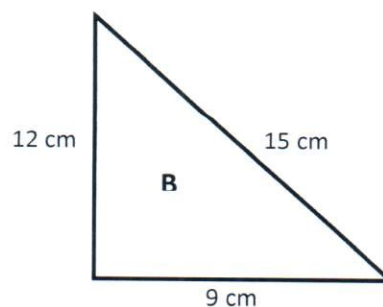
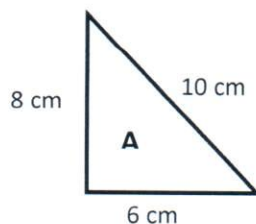
$$SF_{\text{enlargement}} = \frac{\text{BIG}}{\text{SMALL}} = \frac{6}{4} = 1.5$$

Notice that we have used the corresponding lengths. The rectangle **ABCD** is 1.5 times bigger than rectangle **abcd**. We therefore multiply the width of the smaller rectangle with the enlargement scale factor to find the size of the bigger width.

$$CD = 12 \times 1.5 = 18$$

Example 3

Find the scale factor from B to A and from A to B.



Solution:

$$\text{From B to A is a reduction scale factor. Thus, } SF = \frac{\text{SMALL}}{\text{BIG}} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15} = \frac{2}{3}.$$

Similarly, to find the scale factor from A to B, it is an enlargement scale factor. Thus,

$$SF = \frac{\text{BIG}}{\text{SMALL}} = \frac{9}{6} = \frac{12}{8} = \frac{15}{10} = \frac{3}{2} = 1.5$$



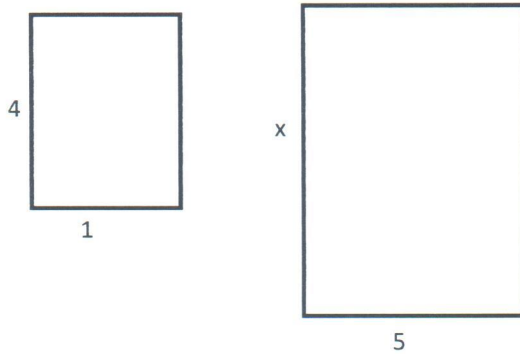
Learning Activity 12.1.1.1



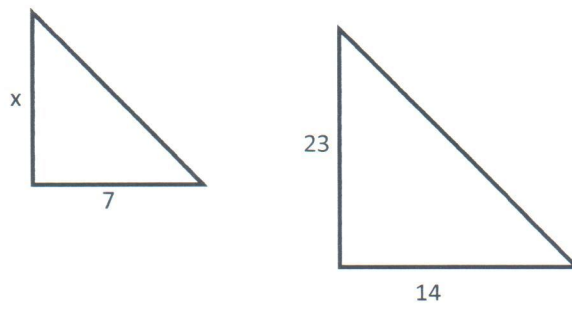
20 minutes

1. For the similar shapes below, find the missing measure.

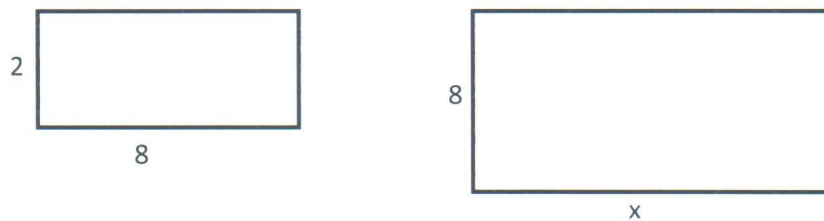
a.



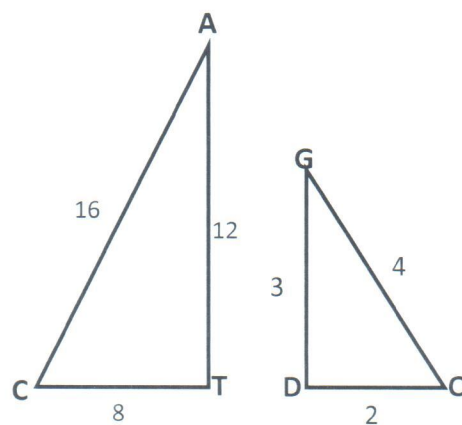
b.



c.

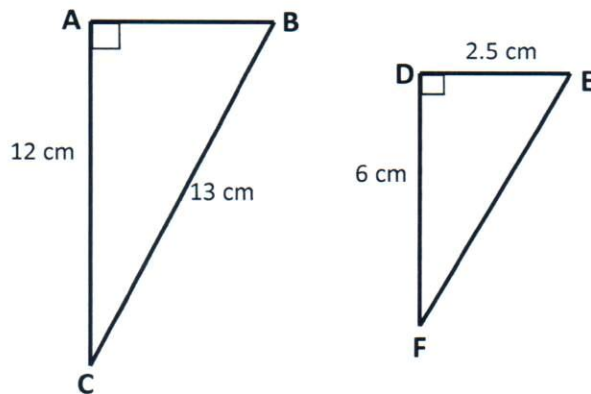


2. What is the scale factor from triangle CAT to triangle DOG?

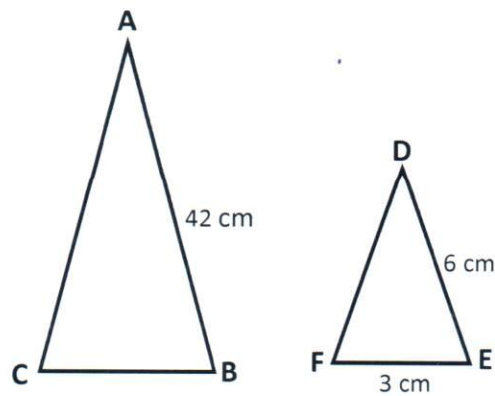




3. The following diagram shows two similar triangles:



- (a) What is the scale factor from triangle **ABC** to triangle **DEF**?
- (b) What is the length of side **EF**?
- (c) What is the length of side **AB**?
4. Two similar isosceles triangles are shown in the diagram below.



- (a) What is the length of side **DF**?
- (b) What is the length of side **AC**?
- (c) What is the scale factor from triangle **DEF** to triangle **ABC**?
- (d) What is the length of side **BC**?



12.1.1.2: Perimeter and Area of Similar Figures

When two figures are similar, the reduced ratio of any two corresponding sides is called the **scale factor** of the similar triangles. In Figure 1, $\triangle ABC \sim \triangle DEF$. ($\sim$ means **similar**)

Perimeter Scale Factor

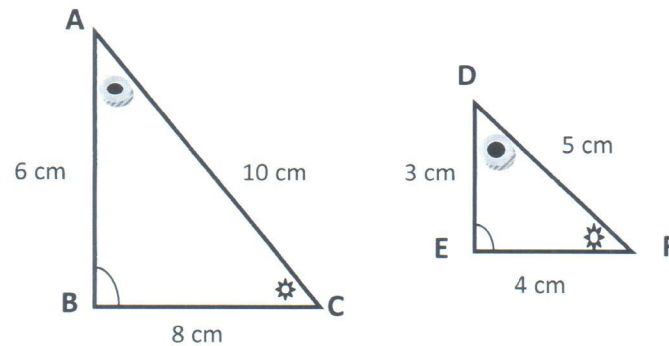


Figure 1: Similar triangles whose scale factor is 2:1

The ratios of corresponding sides are $\frac{6}{3}, \frac{8}{4}, \frac{10}{5}$. These are all reduced to $\frac{2}{1}$. It is then said that the scale factor of these two similar triangles is 2:1.

The perimeter of $\triangle ABC$ is 24 inches ($6 + 8 + 10$), and the perimeter of $\triangle DEF$ is 12 inches ($3 + 4 + 5$). When you compare the ratios of the perimeters of these similar triangles, you also get 2:1.

This leads us to the following theorem:

If two similar triangles have a scale factor of a:b then the ratio of their perimeters is a:b.

Example

In Figure 2, $\triangle ABC \sim \triangle DEF$. Find the perimeter of $\triangle DEF$.

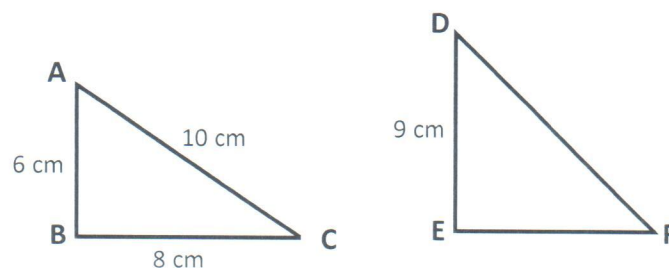


Figure 2: Perimeter of similar triangles.



Solution:

$$\frac{\text{perimeter of } \triangle ABC}{\text{perimeter of } \triangle DEF} = \frac{AB}{DE}$$

$$\frac{24}{\text{perimeter of } \triangle DEF} = \frac{6}{9}$$

$$6(\text{perimeter of } \triangle DEF) = 9(24)$$

$$6(\text{perimeter of } \triangle DEF) = 214$$

$$= \frac{214}{6}$$

$$\text{perimeter of } \triangle DEF = 36 \text{ cm}$$

Area Scale Factor

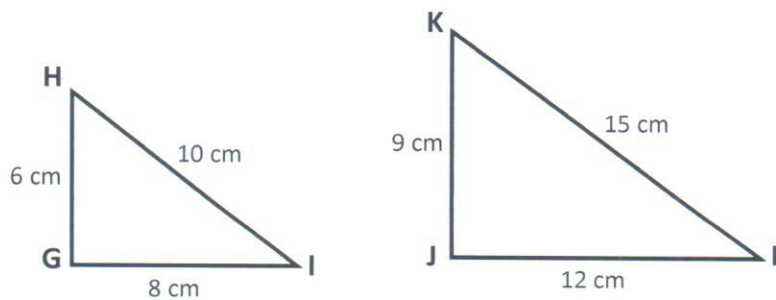


Figure 3: Similar right triangles whose scale factor is 2:3.

Because $\mathbf{GH} \perp \mathbf{GI}$ and $\mathbf{JK} \perp \mathbf{JL}$, they can be considered base and height for each triangle. You can now find the area of each triangle.

$$\text{area } \triangle GHI = \frac{1}{2}(6)(8) = 24 \text{ in}^2$$

$$\text{area } \triangle JKL = \frac{1}{2}(9)(12) = 54 \text{ in}^2$$

Now you can compare the ratio of the areas of these similar triangles.

$$\frac{\text{area } \triangle GHI}{\text{area } \triangle JKL} = \frac{24}{54} = \frac{4}{9} = \frac{2^2}{3^2}$$

This leads us to the following theorem:

If two similar triangles have a scale factor of $a:b$, then the ratio of their areas is $a^2:b^2$.



For example, here are two triangles.



There are simple steps you will take to find the area of the large triangle.

Step 1: Find the linear scale factor.

In general, to find the scale factor, we divide the large quantity with the small quantity.

$$SF_{\text{enlargement}} = \frac{\text{Big}}{\text{Small}} = \frac{6}{4} = 1.5$$

Step 2: Find the area scale factor by squaring the linear scale factor.

$$\begin{aligned}\text{Area Scale Factor} &= (\text{Linear Scale Factor})^2 = \left(\frac{6}{4}\right)^2 \\ &= \frac{36}{16} \\ &= 2.25\end{aligned}$$

Step 3: Multiply the area scale factor with the small area to find the area for the large triangle.

$$\text{Area} = 12 \times 2.25 = 27 \text{ cm}^2$$

The area for the large shape is 27 cm^2

Note:

To find one of the lengths given the areas, you will need to first find the area scale factors and then find its square root to get the scale factor. You would then multiply if finding a length for a large shape or divide if finding the length for a small shape.



Below are more examples on perimeter and area scale factor.

Example 1

In Figure 4, $\triangle PQR \sim \triangle STU$, find the area of $\triangle STU$.

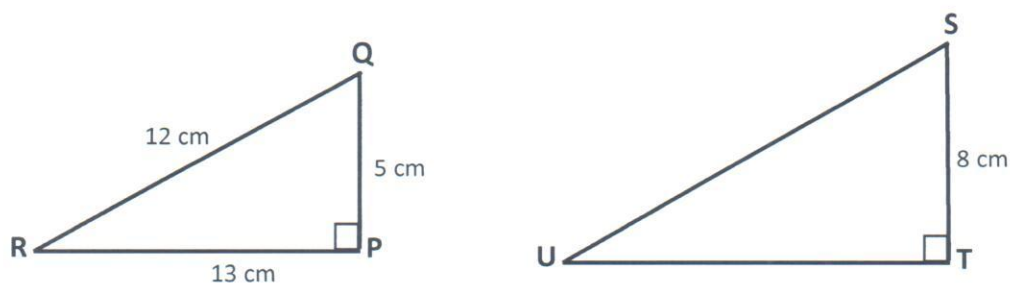


Figure 4

Use the scale factor to determine the relationship between the areas of similar triangles.

Solution:

The scale factor of these similar triangles is 5:8.

$$\frac{\text{area } \triangle PQR}{\text{area } \triangle STU} = \frac{5^2}{8^2} = \frac{25}{64}$$

$$\text{area } \triangle PQR = \frac{1}{2}(13)(5)$$

$$\text{area } \triangle PQR = 32.5 \text{ cm}^2$$

$$\frac{32.5}{\text{area } \triangle STU} = \frac{25}{64}$$

$$25(\text{area } \triangle STU) = 2080 \text{ cm}^2$$

$$\text{area } \triangle STU = \frac{2080}{25}$$

$$= 83 \frac{1}{5} \text{ cm}^2 \text{ or } 83.2 \text{ cm}^2$$

Therefore, the area of $\triangle STU$ is $83 \frac{1}{5} \text{ cm}^2$ or 83.2 cm^2 .



Example 2

The perimeters of two similar triangles are in the ratio 3:4. The sum of their areas is 75 cm^2 . Find the area of each triangle.

Solution:

$$\text{If you call the triangles } \Delta_1 \text{ and } \Delta_2, \frac{\text{perimeter}\Delta_1}{\text{perimeter}\Delta_2} = \frac{3}{4}$$

This also means that the scale factor of these two similar triangles is 3: 4.

Let $3x$ be a side in Δ_1

And $4x$ be the corresponding side in Δ_2

$$\text{Then, } \frac{\text{area}\Delta_1}{\text{area}\Delta_2} = \left(\frac{3x}{4x}\right)^2 = \frac{9x^2}{16x^2}$$

Because the sum of the areas is 75 cm^2 , you get

$$\text{Area } \Delta_1 + \text{Area } \Delta_2 = 9x^2 + 16x^2$$

$$75 = 25 x^2$$

$$3 = x^2$$

$$\text{Therefore, Area } \Delta_1 = 9x^2 = 9(3) = 27 \text{ cm}^2$$

$$\text{Area } \Delta_2 = 16x^2 = 16(3) = 48 \text{ cm}^2$$

Example 3

The areas of two similar triangles are 45 cm^2 and 80 cm^2 . The sum of their perimeters is 35 cm. Find the perimeter of each triangle.

Solution:

Name the two triangles Δ_1 and Δ_2 and let the scale factor of the two similar triangles be $a:b$.

$$\frac{\text{area}\Delta_1}{\text{area}\Delta_2} = \left(\frac{a}{b}\right)^2$$

$$\frac{45}{80} = \left(\frac{a}{b}\right)^2$$

$a:b$ is the reduced form of the scale factor. 9:16 is then the reduced form of the comparison of the perimeters.



Reduced the fraction

$$\frac{9}{16} = \left(\frac{a}{b}\right)^2$$

Take the square roots of both sides.

$$\frac{3}{4} = \frac{a}{b}$$

Let $3x$ be perimeter of Δ_1 and $4x$ be perimeter of Δ_2 . Then

$$3x + 4x = 35$$

The sum of perimeters is 35 cm.

$$7x = 35$$

$$x = 5$$

$$\text{Hence, } 3x = 3(5) = 15$$

$$4x = 4(5) = 20$$

Therefore, the Perimeter $\Delta_1 = 3(5) = 15 \text{ cm}$

Perimeter $\Delta_2 = 4(5) = 20 \text{ cm}$

Example 4

Use the diagram of two similar parallelograms below to answer the following questions:



- What is the scale factor from Parallelogram A to Parallelogram B?
- What is the scale factor from Parallelogram B to Parallelogram A?
- Calculate the area and perimeter for Parallelogram A.
- Calculate the area and perimeter for Parallelogram B
- How did the perimeter change from Parallelogram A to Parallelogram B?
- How did the area change from Parallelogram A to Parallelogram B?



Solution:

- a) Scale factor from A to B is reduction scale factor.

The corresponding sides are 2 cm and 8 cm respectively.

$$\text{The scale factor is } \frac{\text{small}}{\text{big}} = \frac{2}{8} = \frac{1}{4}$$

- b) Scale factor from B to A is an enlargement scale factor.

The corresponding sides are 8 cm and 2 cm respectively.

$$\text{The scale factor is } \frac{\text{big}}{\text{small}} = \frac{8}{2} = 4$$

- c) Area and Perimeter of parallelogram A

$$\text{Area} = bh$$

$$= 8 \times 10$$

$$= 80 \text{ cm}^2$$

$$\text{Perimeter} = 2(a + b) \text{ where } a \text{ and } b \text{ are bases}$$

$$= 2(8 + 11)$$

$$= 38 \text{ cm}$$

- d) Area and perimeter of Parallelogram B

$$\text{Area} = bh$$

$$= 2 \times 2.5$$

$$= 5 \text{ cm}^2$$

$$\text{Perimeter} = 2(a + b) \text{ where } a \text{ and } b \text{ are bases}$$

$$= 2(2 + 2.75)$$

$$= 9.5 \text{ cm}$$

- e) The perimeter of Parallelogram A reduced four times the perimeter of parallelogram B.

That is the scale factor of the perimeters of parallelogram A to B is also $\frac{1}{4}$.

- f) The ratio of area from parallelogram A to parallelogram B is $\frac{1}{16}$ is the square of the scale factor of parallelogram A to parallelogram B.



Example 5

A shape has an area of 5 cm^2 . Its lengths are enlarged by scale factor 5. What is the area of the new shape?

Solution: Area of original shape = 5 cm^2 ; Area of new shape = x , $SF_{\text{enlargement}} = 5$

$$\frac{\text{Area}(x)}{\text{Area}(a)} = (SF)^2$$

$$\frac{\text{Area}(x)}{5} = 5^2$$

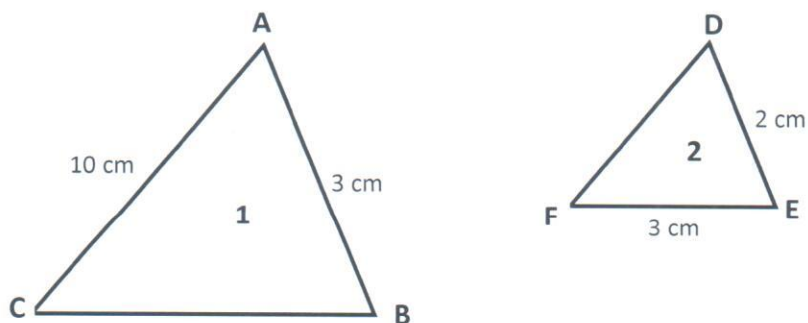
$$\text{Area}(x) = 25 \times 5 = 125 \text{ cm}^2$$

Now do the learning activity.

**Learning Activity 12.1.1.2**

20 minutes

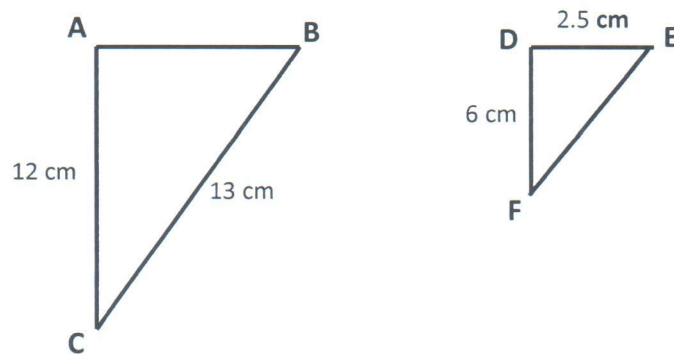
1. Refer to the figure to answer the following questions.



- What is the scale factor from Triangle 1 to Triangle 2?
- What is the length of side DF?
- What is the scale factor from Triangle 2 to Triangle 1?
- What is the length of side BC?



2. The following diagram shows two similar triangles:

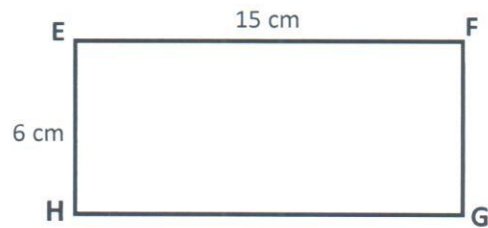
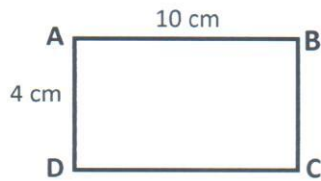


- What is the scale factor from triangle ABC to triangle DEF?
 - What is the length of side EF?
 - What is the length of side AB?
3. The perimeter of Figure A is 9 m. The perimeter of a similar Figure B is 15 m. What is the ratio of the perimeter of A to the perimeter of B?
4. Triangle A is similar to Triangle B. If the scale factor of ΔA to ΔB is 4 to 5,
- What is the ratio of the perimeters of ΔA to ΔB ?
 - What is the ratio of the areas of ΔA to ΔB ?
5. A square has an area of 4 cm^2 . Its sides are enlarged by scale factor 4.
What is the length of the sides of the enlarged square?

**SUMMATIVE TASK 12.1.1**

20 minutes

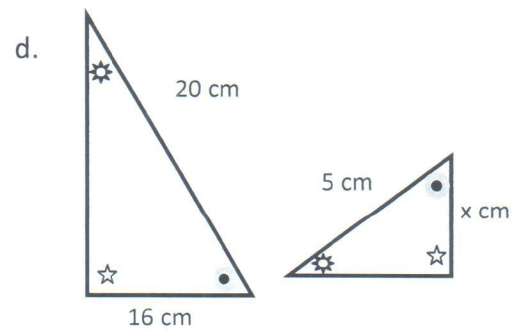
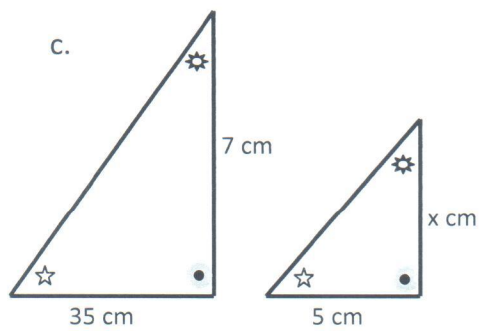
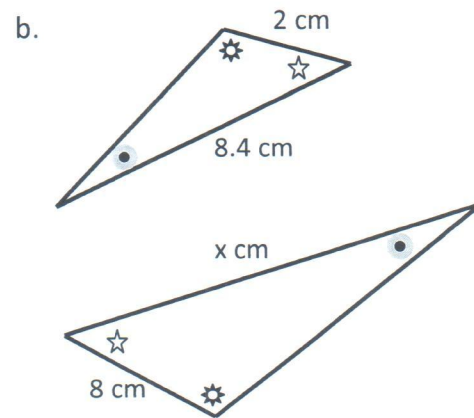
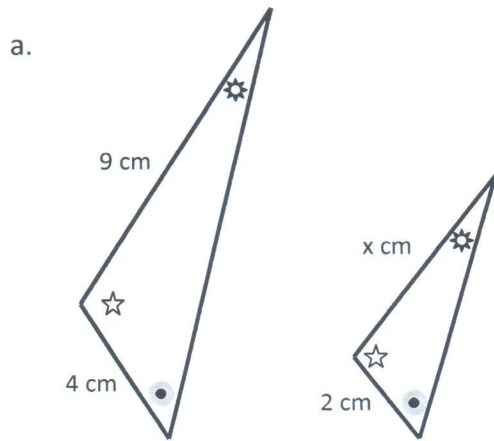
1. Use the figures below to answer questions a to f.



- Find the scale factor of the sides. (enlargement)
- Find the perimeter of ABCD.
- Find the perimeter of EFGH.
- Find the area of ABCD.
- Find the area of EFGH.
- How does the scale factor of the sides compare to the scale factor of the area?

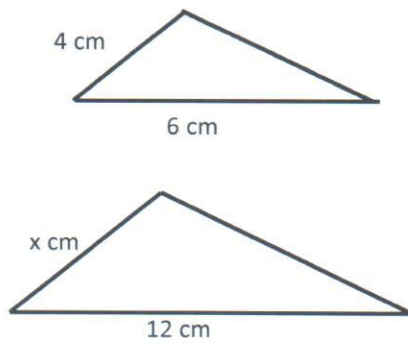


2. Write the scale factors and values for x for the pairs of similar triangles.

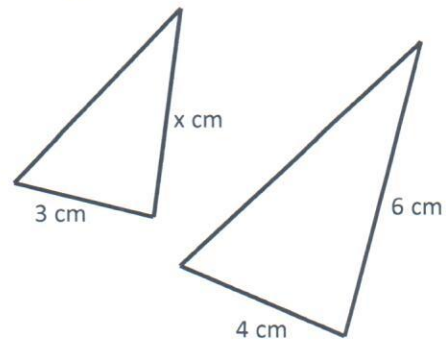




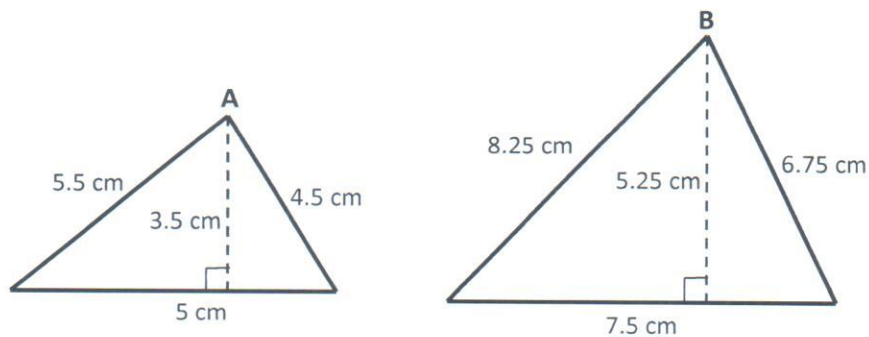
e.



f.



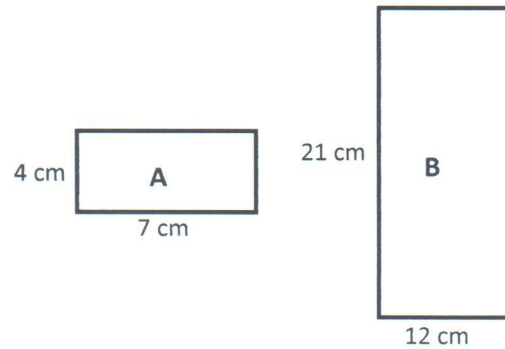
3. Use the diagram of two similar triangles below to answer the following questions.



- a) What is the scale factor from Triangle A to Triangle B? _____
- b) What is the scale factor from Triangle B to Triangle A? _____
- c) Calculate the area and perimeter for Triangle A: Area: _____ Perimeter: _____
- d) Calculate the area and perimeter for Triangle B: Area: _____ Perimeter: _____
- e) How did the perimeter change from Triangle A to Triangle B? _____
- f) How did the area change from Triangle A to Triangle B? _____



4. Use the diagram of two similar rectangles below to answer the following questions:



- a) What is the scale factor from Rectangle A to Rectangle B? _____
- b) What is the scale factor from Rectangle B to Rectangle A? _____
- c) Calculate the area and perimeter for Rectangle A: A: _____ P: _____
- d) Calculate the area and perimeter for Rectangle B: A: _____ P: _____
- e) How did the perimeter change from Rectangle A to Rectangle B? _____
- f) How did the area change from Rectangle A to Rectangle B? _____
5. The scale factor of two similar polygons is 2:3. The perimeter of the larger polygon is 150 centimetres. What is the perimeter of the smaller polygon?
6. A square has an area of 9 cm^2 . Its sides are enlarged by scale factor 5. What is the length of the sides of the enlarged square?
7. A shape has an area of 5 cm^2 . Its lengths are enlarged by scale factor 4. What is the area of the new shape?



12.1.2: PERIMETER AND AREA OF TRIANGLES

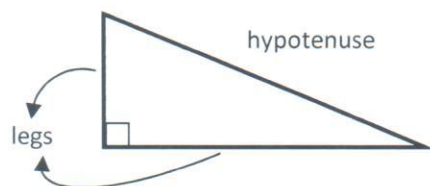
Perimeter and **area** are two important and fundamental mathematical topics. They help you to quantify physical space and also provide a foundation for more advanced mathematics found in algebra, trigonometry, and calculus. Perimeter is a measurement of the distance around a shape and area gives us an idea of how much surface the shape covers.

Knowledge of area and perimeter is applied practically by people every day, such as architects, engineers, and graphic designers, and in mathematics. Understanding how much space you have and learning how to fit shapes together exactly will help you when you want to paint a room, buy a house, renovate a kitchen, or build a deck.

12.1 2.1: Pythagorean Theorem

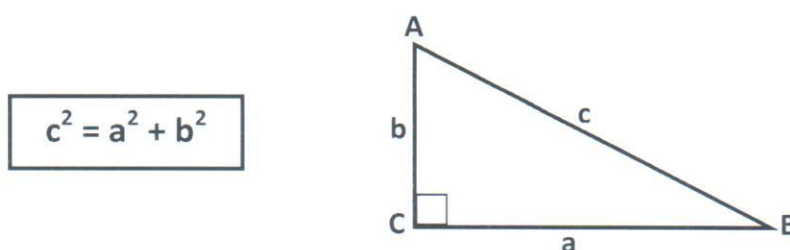
A right angled triangle is a triangle which has a right angle as one of its angles.

The side opposite the right angle is called the **hypotenuse** and is the longest side of the triangle. The other two sides are called **legs** of the triangle.



In any right angled triangle, the square of the hypotenuse c is equal to the sum of the squares of the other two sides a and b .

In formula:



By using the Pythagorean Formula,

- we can find the missing side of a right triangle
- we can verify if a triangle is right or not.

Now look at the examples on how the Pythagorean formula is used on the next page.



Example 1 Find the length of the hypotenuse in the triangle below. Write the answer to 2 decimal places.

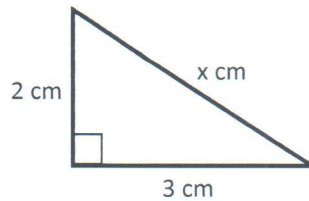
Solution:

$$c^2 = a^2 + b^2$$

$$x^2 = 2^2 + 3^2$$

$$x^2 = 4 + 9 = 13$$

$$x = \sqrt{13} \text{ cm or } 3.61 \text{ cm}$$



Example 2 Find the length of the third side labelled x, of the triangle below.

Solution:

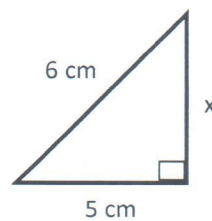
$$a^2 + b^2 = c^2$$

$$x^2 + 5^2 = 6^2$$

$$x^2 = 36 - 25$$

$$x^2 = 11$$

$$x = \sqrt{11} \text{ cm or } 3.32 \text{ cm}$$



Example 3 Find x in the triangle below.

Solution:

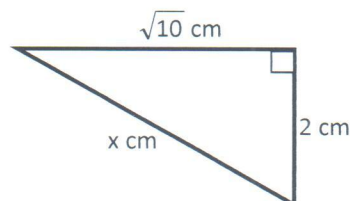
$$c^2 = a^2 + b^2$$

$$x^2 = 2^2 + 10^2$$

$$x^2 = 4 + 10$$

$$x^2 = 14$$

$$x = \sqrt{14} \text{ cm or } 3.74 \text{ cm}$$



Example 4 Find the value of x in the given triangle.

Solution:

$$c^2 = a^2 + b^2$$

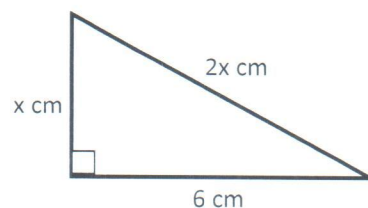
$$(2x)^2 = x^2 + 6^2$$

$$4x^2 = x^2 + 36$$

$$3x^2 = 36$$

$$x^2 = 12$$

$$x = \sqrt{12} \text{ cm or } 3.46 \text{ cm}$$





Example 5 Determine if a triangle with sides 9m, 11m and 14m is a right-angled triangle.

Solution: Use the Pythagorean Formula to verify.

$$\begin{aligned}c^2 &= a^2 + b^2 \\14^2 &= 9^2 + 11^2 \\196 &= 81 + 121 \\196 &\neq 202\end{aligned}$$

Therefore, the triangle is **not** a right-angled triangle.

Problem Solving Using Pythagorean Theorem

Many practical problems involve triangles. We can apply Pythagoras' theorem to any triangle that is right angled. We can also use the theorem to test whether a right triangle exists.

Points to remember when solving word problems:

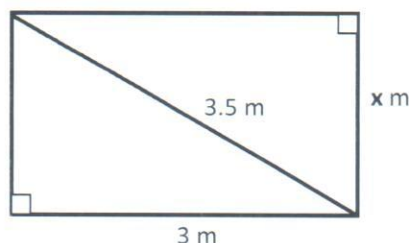
- Draw a clear, neat diagram of the situation.
- Mark and label known lengths and right angles.
- Use a symbol such as x to represent the unknown length.
- Write down the Pythagoras theorem for the given information.
- Solve the equation.
- Where necessary, write your answer in sentence form.

Example 1

A rectangular gate is 3 m wide and has a diagonal of 3.5 m. How high is the gate?

Solution: Let x be the height of the gate.

Now, we draw the diagram.



Using the Pythagorean Theorem:

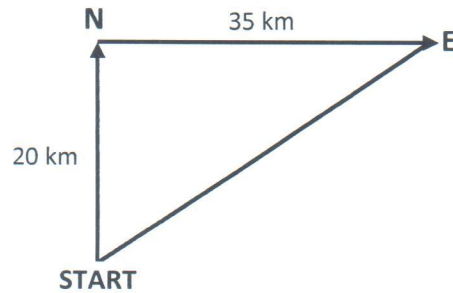
$$\begin{aligned}(3.5)^2 &= x^2 + 3^2 \\12.25 &= x^2 + 9 \\12.25 - 9 &= x^2 \\3.25 &= x^2 \\\sqrt{3.25} &= x \\1.80 &= x \\ \text{Or } x &= 1.80 \text{ m}\end{aligned}$$

Thus, the gate is approximately **1.80 m** high.



Example 2

A ship sails 20 km due North and then 35 km due East. How far is it from the starting point?



Solution: Let d = be the distance of the ship from the starting point to East.

$$d^2 = (20)^2 + (35)^2$$

$$d = \sqrt{400 + 1225}$$

$$d = \sqrt{1625}$$

$$d = 40.31$$

$$d = 40 \text{ km (to the nearest kilometre)}$$

Therefore, the distance is approximately 40 kilometres from the starting point.

Example 3

A 4 m ladder leans against a vertical wall with its foot 2 m from the wall. How far up the wall does the ladder reach?

Solution: Using the Pythagorean Formula

$$c^2 = a^2 + b^2$$

$$4^2 = 2^2 + b^2$$

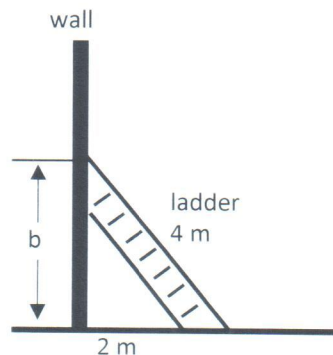
$$16 = 4 + b^2$$

$$16 - 4 = b^2$$

$$12 = b^2$$

$$\sqrt{12} = b$$

$$b = 3.5 \text{ m}$$



Therefore, the ladder reaches the wall at approximately 3.5 metres.

Now do the learning activity on the next page.



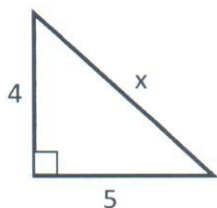
Learning Activity 12.1.2.1



20 minutes

1. Find the value of all pronumerals correct to 1 decimal place.

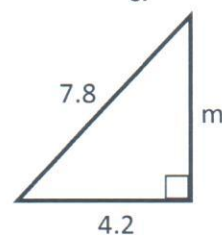
a.



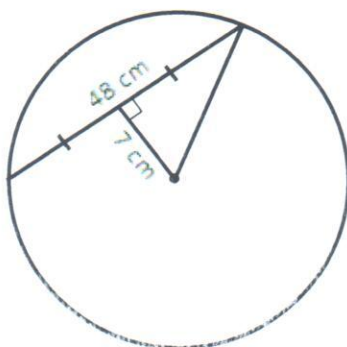
b.



c.

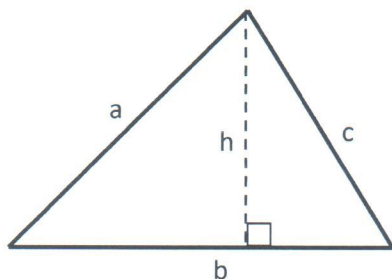


2. A ladder 10 m long is leaning against a wall. The foot of the ladder is 6 m from the base of the wall. At what height does the ladder rest on the wall?
3. A yacht sails 5 km due west and then 8 km due south. How far is it from its starting point?
4. Use Pythagorean Theorem to work out whether this triangle with sides 13, 15 and 17 is a right angled triangle.
5. In a circle, a chord of 48 cm is 7 cm from the centre. Calculate the area of the circle.



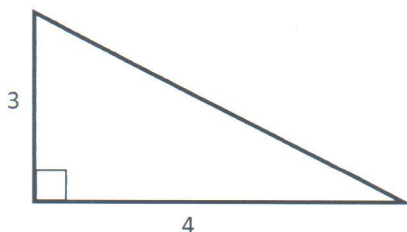
**12.1.2.2: Finding the Area of Triangles using the Sine Ratio**

The conventional formula for the area of a triangle is $A = \frac{1}{2}bh$, where **b** is the length of the base and **h** is the height. This method is the basic way of finding the area of a triangle given the base and the height of the triangle.



Example

Calculate the area of the right triangle with legs measuring 3 cm and 4 cm.



Solution: Using the formula: $A = \frac{1}{2}bh$

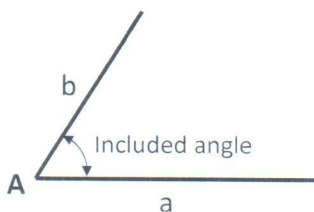
$$A = \frac{1}{2}(4)(3)$$

$$A = 6 \text{ cm}^2$$

Trigonometry, however, provides additional ways to find the area of a triangle using the trigonometric functions.

Knowing Two Sides and the Included Angle

When we know **two sides** and **the included angle (SAS)**, there is another formula (in fact three equivalent formulas) we can use.



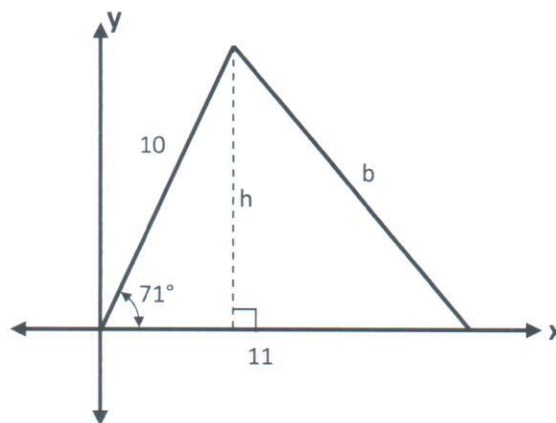


If two sides and their included angle are given, the **sine ratio** can be used to calculate the **height** of the triangle. After finding the height, the formula is used.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

Example

Find the area of the triangle in the figure below.



Solution:

The base of the triangle is 11. You can find the height of the triangle by using the fact that

$$\sin 71 = \frac{h}{10} \quad (\text{Use your calculator } \sin 71 = 0.9455)$$

$$0.9455 = \frac{h}{10}$$

$$h = (10)(0.9455)$$

$$h = 9.455 \text{ units}$$

Therefore, $A = \frac{1}{2}bh$

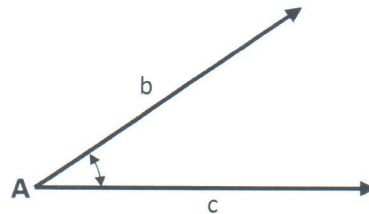
$$A = \frac{1}{2}(11)(9.455)$$

$$A = 52 \text{ square units}$$



- 2) Another way of finding the area given two sides and the included angle is by using the formula:

$$\text{Area} = \frac{1}{2} ab \sin(C)$$



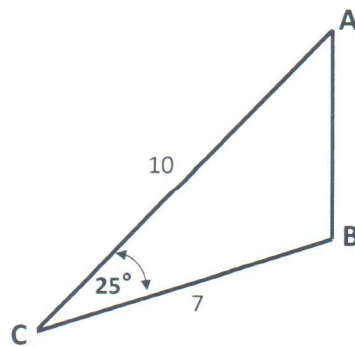
Where b and c are the sides and $\angle A$ is the included angle (included means in between). Depending on which sides and angles we know, the formula can be written in three ways:

Either $\text{Area} = \frac{1}{2} ab \sin C$ or $\text{Area} = \frac{1}{2} bc \sin A$ or $\text{Area} = \frac{1}{2} ac \sin B$

They are the same formula, just with the sides and angle changed.

Example 1

Find the area of this triangle.



Solution:

We know $\angle C = 25^\circ$, and sides $a = 7$ and $b = 10$.

So using the formula: $\text{Area} = \frac{1}{2} ab \sin C$

Put in the values we know: $\text{Area} = \frac{1}{2} (10)(7) \sin 25$

$$= 35 (0.4226)$$

$$= 14.8 \text{ sq. units (to 1 decimal place)}$$

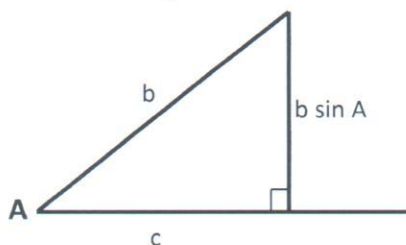
**How to Remember**

Just think "**abc**": $\text{Area} = \frac{1}{2} ab \sin C$

How does it work?

Well, we know that we can find an area when we know the base and height:

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$



In this triangle:

- the base is c
- the height is $b \times \sin A$

Putting that together gives us: $\text{Area} = \frac{1}{2} \times (c) \times (b \times \sin A)$

This is simply: $\text{Area} = \frac{1}{2} bc \sin A$

By changing the labels on the triangle we can also get:

i. $\text{Area} = \frac{1}{2} ab \sin C$

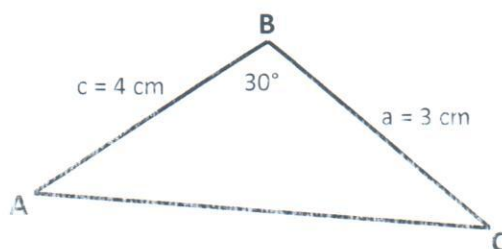
ii. $\text{Area} = \frac{1}{2} ac \sin B$

Example 2

Find the area of the triangle shown below.

Solution:

$$\begin{aligned}\text{Area} &= \frac{1}{2} ac \sin B \\ &= \frac{1}{2} (3)(4) \sin 30^\circ \\ &= 6(0.5) \\ &= 3 \text{ cm}^2\end{aligned}$$





Example 3

Find the area of the obtuse angled triangle shown below.



Solution:

Given $C = 140^\circ$; $a = 105$ mm; $b = 75$ mm

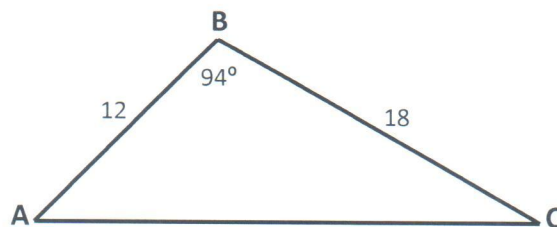
$$\begin{aligned}\text{Area} &= \frac{1}{2} ab \sin C \\ &= \frac{1}{2} (105)(75) \sin 140^\circ \\ &= \frac{1}{2} (105)(75) 0.6428 \\ &= 2531 \text{ mm}^2\end{aligned}$$

Now do the learning activity.

**Learning Activity 12.1.2.2**

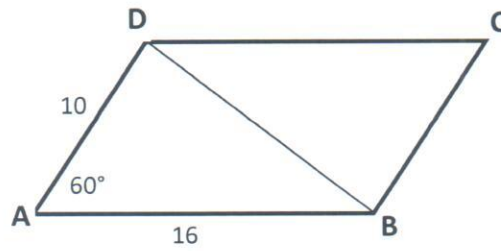
20 minutes

1. Given the triangle below, find its area. Express the area rounded to three decimal places.

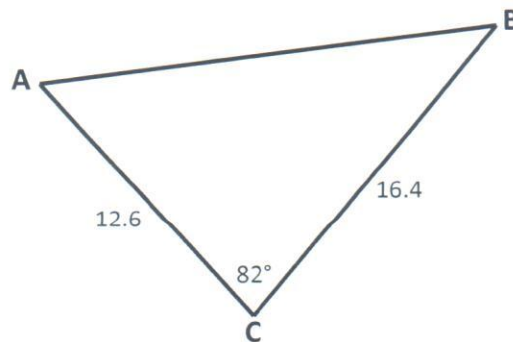




2. Given the parallelogram shown at the right, find its EXACT area.

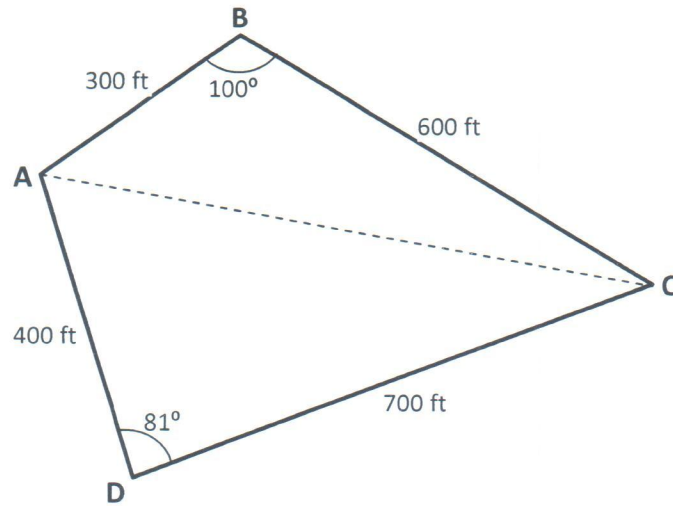


3. What is the area of the triangle? Give your answer in square units correct to 1 decimal place.

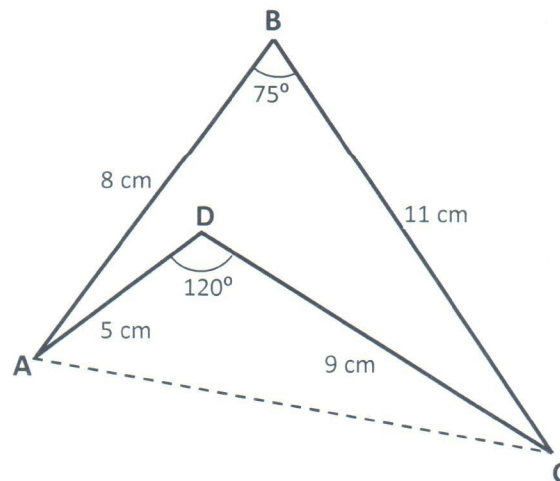




4. What is the area of the field (to the nearest square foot)?



5. What is the area of the logo ABCD?



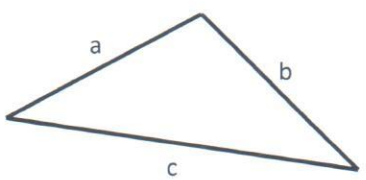
**12.1.2.3: The Heron's Formula**

Heron's formula is named after Hero of Alexandria, a Greek engineer and mathematician from 10 to 70 AD. You can use this formula to find the area of a triangle using the three side lengths.

Therefore, you do not have to rely on the formula for area that uses base and height. The figure below illustrates the general formula where **S** represents the semi-perimeter of the triangle.

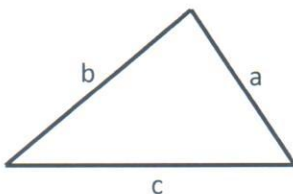
Heron's Formula

$$s = \frac{a+b+c}{2}$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$


Knowing Three Sides

When three sides of a triangle are known, then we use this two-step process to find the area.



Step 1 Calculate "s" (Half of the triangle's perimeter). $s = \frac{a+b+c}{2}$

Step 2 Calculate the area. $\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$

Example 1

What is the area of a triangle where every side is 5 units long?

Solution: Step 1: $s = \frac{a+b+c}{2}$

$$s = \frac{5+5+5}{2} = 7.5$$

Step 2: $\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$

$$\text{Area} = \sqrt{7.5(2.5)(2.5)(2.5)}$$

$$A = \sqrt{117.1875}$$

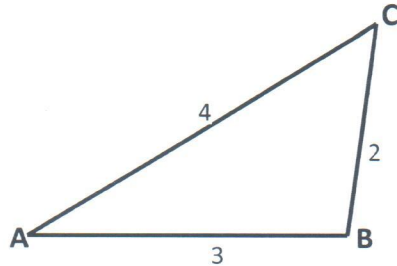
$$A = 10.8253175$$

$$A = \mathbf{10.82 \text{ square units}}$$
 (rounded off to 2 decimal places)



Example 2

Use Heron's Formula to find the area of triangle ABC, if $AB = 3$, $BC = 2$ and $CA = 4$.



Solution:

Step 1: Calculate the semi perimeter, S .

$$S = \frac{a+b+c}{2}$$
$$S = \frac{3+4+2}{2} = 4.5 \text{ units}$$

Step 2: Substitute S into the formula. Round off answer to the nearest tenth.

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{Area} = \sqrt{4.5(2.5)(1.5)(0.5)}$$

$$A = \sqrt{8.4375} = 2.9 \text{ units}^2$$

Since Heron's formula relates the side lengths, perimeter and area of a triangle, you might need to answer questions in the following example.

Example 3

Given a triangle with an area of 8.94 square centimetres, a perimeter of 16 cm and side lengths $AB = 3$ cm and $CA = 7$ cm, what is the length of side BC ?

Solution:

Step 1: Calculate the semi perimeter, S .

$$S = \frac{\text{perimeter}}{2} = \frac{16}{2} = 8 \text{ cm}$$

Step 2: Substitute known values into the formula. Let x equal side length BC .

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{Area} = \sqrt{s(s-AB)(s-BC)(s-CA)}$$

$$8.94 = \sqrt{8(8-3)(8-x)(8-7)}$$



Step 3: Solve for x (Square both sides and go from there.)

$$8.94^2 = \left(\sqrt{8(8-3)(8-x)(8-7)} \right)^2$$

$$79.9236 = 8(5)(8-x)(1)$$

$$79.9236 = 40(8-x)$$

$$\frac{79.9236}{40} = 8-x$$

$$1.999809 = 8-x$$

$$2 = 8-x \quad (1.999809 \approx 2)$$

$$x = 8-2$$

$$x = 6.0 \text{ cm}$$

Therefore, the length of side BC is 6 cm.

Now do the learning activity.



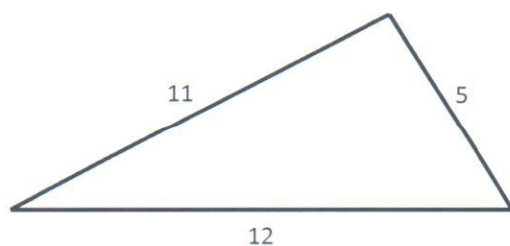
Learning Activity 12.1.2.3



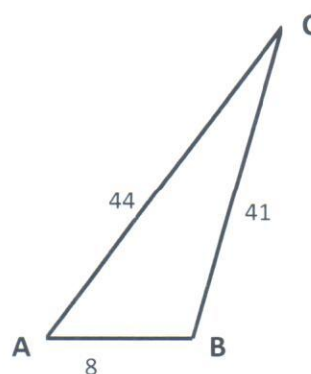
20 minutes

1. Calculate the area of the triangle using Heron's formula.

a.



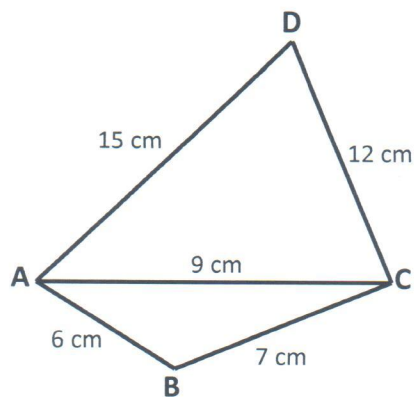
b.



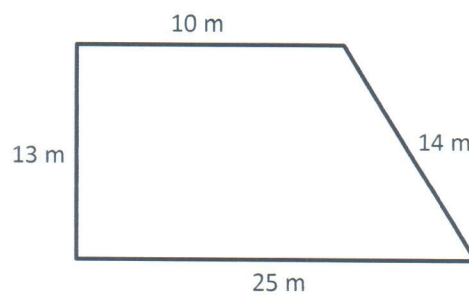
2. Use Heron's formula to find the area of a triangle of lengths 2, 3 and 3. Give your answer in surd form.



3. Find the area of quadrilateral **ABCD**, whose sides $\overline{AB} = 6$ cm, $\overline{BC} = 7$ cm, $\overline{CD} = 12$ cm, $\overline{AD} = 15$ cm and $\overline{AC} = 9$ cm.



4. A field is in the shape of a trapezium whose parallel sides are 25 m and 10 m. The non-parallel sides are 14 m and 13 m. Find the area of the field.



5. An umbrella is made by stitching 10 triangular pieces of cloth of two different colors red and blue. Each piece is measuring 20 cm, 50 cm and 50 cm. How much cloth of each color is required for the umbrella?





12.1.2.4: The Perimeter and Areas of Polygons

To solve problems in measurement of everyday objects, you need to know the formula for the perimeter, area, surface area and volume of different shapes. In this section, we will recall what you have studied previously about perimeter and areas of polygons.

Perimeter

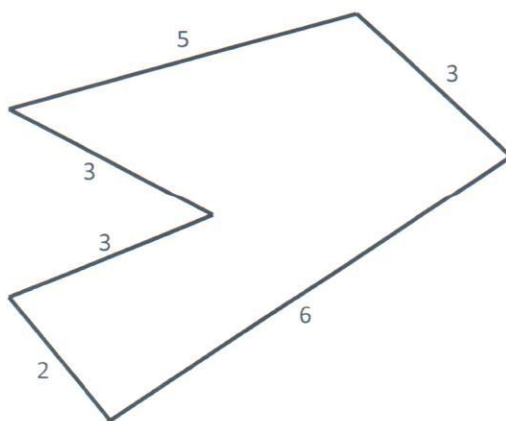
The perimeter of a two-dimensional shape is the distance around the shape.

You can think of wrapping a string around a triangle. The length of this string would be the perimeter of the triangle. Or walking around the outside of a park, you walk the distance of the park's perimeter. Some people find it useful to think "peRIMeter" because the edge of an object is its rim and peRIMeter has the word "rim" in it.

If the shape is a polygon, then you can add up all the lengths of the sides to find the perimeter. Make sure that all the lengths are measured in the same units. You measure perimeter in linear units, which is one dimensional. Examples of units of measure for length are centimetres (cm), millimetres (mm) or metres (m).

Example 1

Find the perimeter of the given figure. All measurements indicated are centimetres.



Solution:

$$P = 5 + 3 + 6 + 2 + 3 + 3$$

Since all the sides are measured in centimetres, just add the lengths of all six sides to get the perimeter.

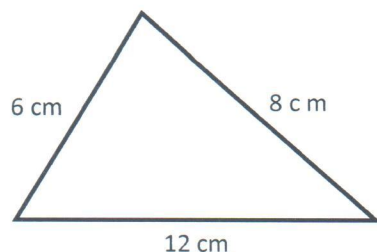
$$P = 22 \text{ centimetres}$$

Remember to include units.

This means that a tightly wrapped string running the entire distance around the polygon would measure 22 cm long.

**Example 2**

Find the perimeter of a triangle with sides measuring 6 cm, 8 cm, and 12 cm.



Solution:

$$P = 6 \text{ cm} + 8 \text{ cm} + 12 \text{ cm}$$

Since all the sides are measured in centimetres, just add the lengths of all three sides to get the perimeter.

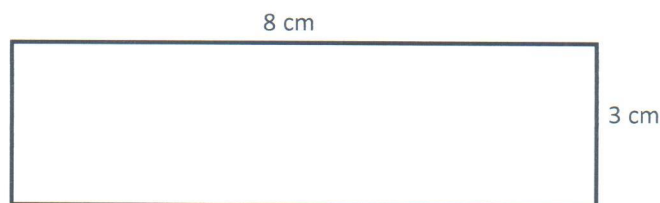
$$P = 26 \text{ cm}$$

Sometimes, you need to use what you know about a polygon in order to find the perimeter.

Let us look at the rectangle in the next example.

Example 3

A rectangle has a length of 8 centimetres and a width of 3 centimetres. Find the perimeter.



Solution:

$$P = (3 + 3 + 8 + 8) \text{ cm}$$

Since this is a rectangle, the opposite sides have the same lengths, 3 cm and 8 cm. Add up the lengths of all four sides to find the perimeter.

$$P = 22 \text{ cm}$$

Notice that the perimeter of a rectangle always has two pairs of equal length sides. In the above example you could have also written $P = 2(3) + 2(8) = 6 + 16 = 22 \text{ cm}$.

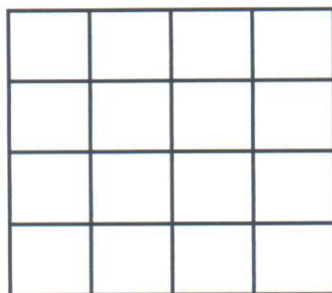
The formula for the perimeter of a rectangle is often written as $P = 2L + 2W$, where **L** is the length of the rectangle and **W** is the width of the rectangle.



AREA

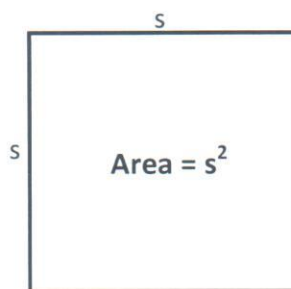
The **area** of a two-dimensional figure describes the amount of surface the shape covers.

You measure area in square units of a fixed size. Examples of square units of measure are square centimeters, square metres or square miles. When finding the area of a polygon, you count how many squares of a certain size will cover the region inside the polygon. Let us look at a 4 x 4 square.



You can count that there are 16 squares, so the area is 16 square units. Counting out 16 squares does not take too long, but what about finding the area if this is a larger square or the units are smaller? It could take a long time to count.

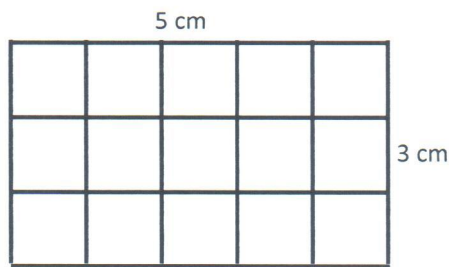
Fortunately, you can use multiplication. Since there are 4 rows of 4 squares, you can multiply $(4)(4)$ to get 16 squares. And this can be generalized to a formula for finding the area of a square with any length, s : **Area = $(s)(s) = s^2$**



You can write " cm^2 " for square centimetre and " m^2 " for square metre.

To help you find the area of the many different categories of polygons, mathematicians have developed formulas. These formulas help you find the area more quickly than by simply counting. The formulas you are going to look at are all developed from the understanding that you are counting the number of square units inside the polygon.

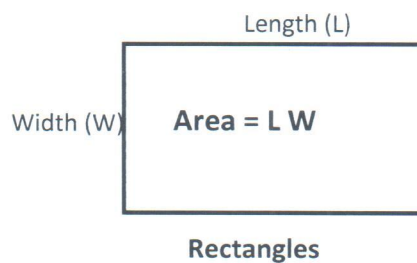
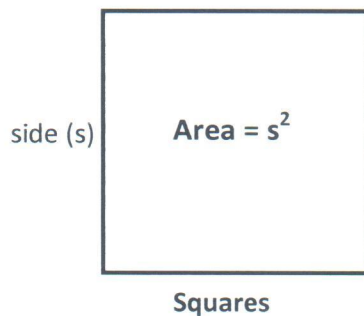
Let us look at a rectangle on the next page.



You can count the squares individually, but it is much easier to multiply 3 times 5 to find the number more quickly. And more generally, the area of any rectangle can be found by multiplying the length and the width.

Most common shapes have a formula we can use to find the area of that shape.

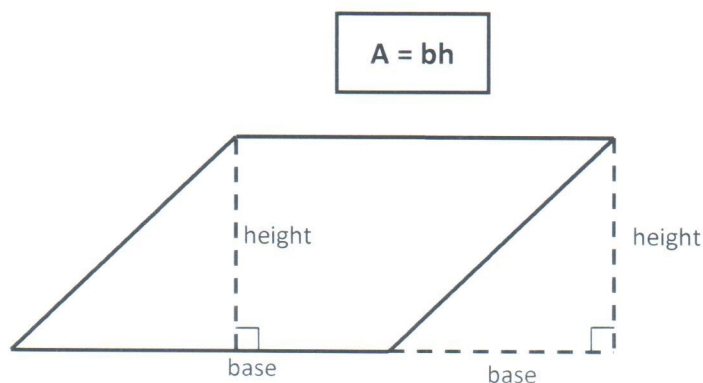
Area of Parallelograms



The formula for the area of any parallelogram (remember, a rectangle is a type of parallelogram) is the same as that of a rectangle:

$$\text{Area} = L \times W \text{ or } \text{Area} = LW$$

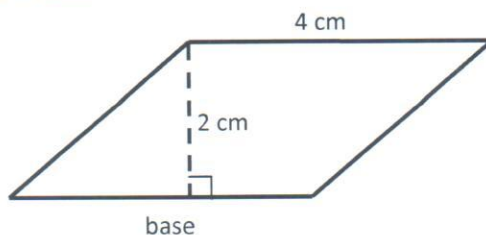
Notice in a rectangle, the length and the width are perpendicular. This should also be true for all parallelograms. **Base (b)** for the length (of the base), and **height (h)** for the width of the line perpendicular to the base is often used. So the formula for the area of a parallelogram is generally written,





Example

Find the area of the parallelogram.



Solution: $A = bh$

Start with the formula for the area of a parallelogram:

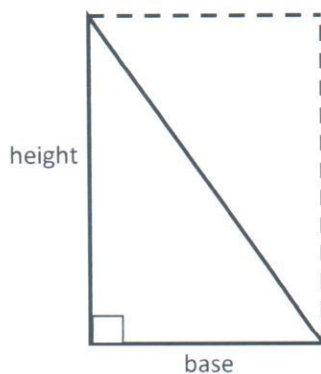
$A = (4 \text{ cm})(2 \text{ cm})$ Substitute the values into the formula

$A = 8 \text{ cm}^2$ Multiply

Area of Triangles and Trapezoids

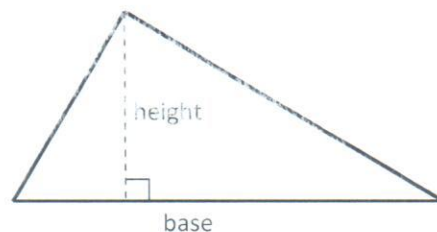
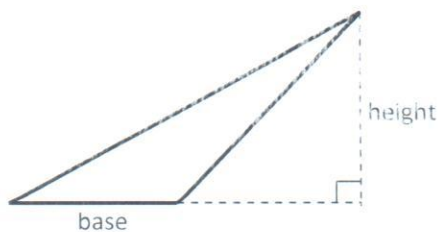
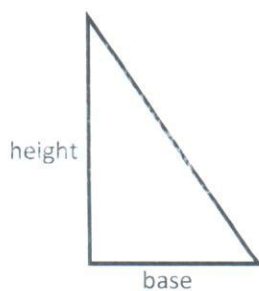
The formula for the area of a triangle can be explained by looking at a right triangle.

Study the image below: A rectangle with the same height and base is the original triangle. The area of the triangle is one half of the rectangle.



Since the area of two congruent triangles is the same as the area of a rectangle, you can come up with the formula **Area** = $\frac{1}{2}bh$ to find the area of a triangle.

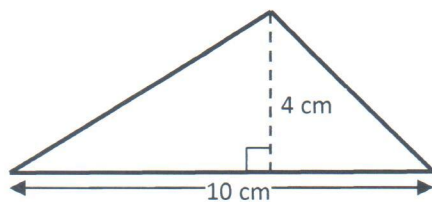
When you use the formula for a triangle to find its area, it is important to identify the base and its corresponding height, which is perpendicular to the base.





Example

A triangle has a height of 4 inches and a base of 10 inches. Find the area.



Solution:

$$A = \frac{1}{2}bh$$

Start with the formula for the area of a triangle.

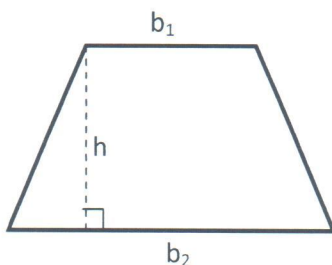
$$A = \frac{1}{2}(10 \text{ cm})(4 \text{ cm})$$

Substitute 10 for the base and 4 for the height.

$$A = \frac{1}{2}(40 \text{ cm}^2)$$

$$A = 20 \text{ cm}^2$$

Now let us study the trapezoid.



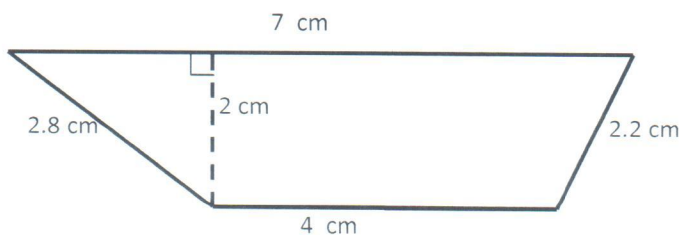
To find the area of a trapezoid, take the average length of the two bases (parallel sides) and multiply that length by the height.

$$A = \frac{(b_1 + b_2)}{2} \times h$$

An example is provided below. Notice that the height of a trapezoid will always be perpendicular to the bases (just like when you find the height of a parallelogram).

Example

Find the area of the trapezoid.





Solution:

$$A = \frac{(b_1 + b_2)}{2} \times h$$

Start with the formula for the area of a trapezoid.

$$A = \frac{(4 \text{ cm} + 7 \text{ cm})}{2} (2 \text{ cm})$$

Substitute 4 cm and 7 cm for the bases and 2 cm for the height, and find A.

$$A = \frac{11 \text{ cm}}{2} (2 \text{ cm}).$$

$$A = 11 \text{ cm}^2$$

The area of the trapezoid is 11 cm^2

Applications of area and perimeter of polygons

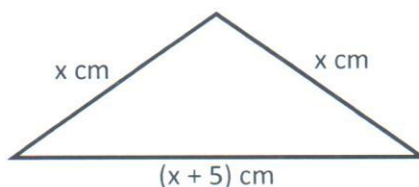
Example 1

A triangle has a perimeter of 50 cm. If two of its sides are equal and the third side is 5 cm more than the equal sides, what is the length of the third side?

Solution:

Step 1: Assign variables. Let x = length of the equal side

Sketch the figure.



Step 2: Write out the formula for perimeter of triangle.

$$P = \text{sum of the three sides or } P = a + b + c$$

Step 3: Plug in the values from the question and from the sketch.

$$50 \text{ cm} = x + x + x + 5$$

Combine like or similar terms.

$$50 \text{ cm} = 3x + 5$$

Isolate variable x .

$$3x = 50 - 5$$

$$3x = 45$$

$$x = 15 \text{ cm}$$

Note that the question requires the length of the third side.

The length of third side = $15 + 5 = 20 \text{ cm}$

Therefore, the length of third side is 20 cm.



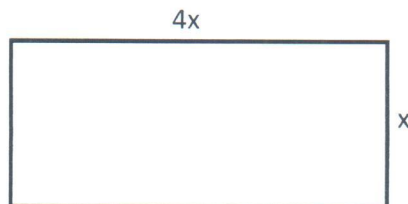
Example 2

A rectangle is 4 times as long as its width. If the length is increased by 4 inches and the width is decreased by 1 inch, the area will be 60 square inches. What were the dimensions of the original rectangle?

Solution:

Step 1: Assign variables.
Sketch the figure

Let x = original width of rectangle



Step 2: Write out the formula for area of rectangle.

$$A = lw$$

Step 3: Substitute the values from the question and from the sketch.

$$60 = (4x + 4)(x - 1)$$

Use distributive property to remove brackets.

$$60 = 4x^2 - 4x + 4x - 4$$

Simplify then express in Quadratic Form.

$$60 = 4x^2 - 4$$

$$4x^2 - 4 - 60 = 0$$

$$4x^2 - 64 = 0$$

Factorize difference of two squares.

$$(2x - 8)(2x + 8) = 0$$

Use the Zero Product Or Null Factor Law.

$$2x - 8 = 0; 2x + 8 = 0$$

We get two values for x .

$$x = 4; \quad x = -4$$

Since x is a dimension, it would be positive. So, we take $x = 4$.

The question requires the dimensions of the original rectangle.

The width of the original rectangle is 4 inches.

The length is 4 times the width $= 4 \times 4 = 16$ inches.

Therefore, the dimensions of the original rectangle are 4 inches and 16 inches.

On the next page is a summary of the Perimeter and Area formulas.

**Perimeter and Area Formulas****square: Perimeter = $4s$**

Area = s^2

rectangle: Perimeter = $2L + 2W$

Area = $L \times W$

parallelogram: Perimeter = sum of all the sides

Area = bh

triangle: Perimeter = $s_1 + s_2 + s_3$

Area = $\frac{1}{2}bh$

trapezoid: Perimeter = sum of all the sides

$A = \frac{(b_1 + b_2)}{2}h$

Area of any polygon = Area of triangle multiplied by the number of triangles.**Working with Perimeter and Area of an Irregular Shaped Polygon**

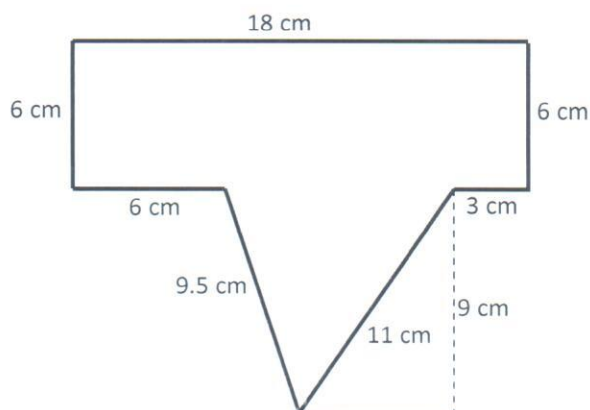
Often you need to find the area or perimeter of a shape that is not a standard polygon. Artists and architects, for example, usually deal with complex shapes. However, even complex shapes can be thought of as being composed of smaller, less complicated shapes, like rectangles, trapezoids and triangles.

To find the perimeter of non-standard shapes, you still find the distance around the shape by adding together the length of each side.

Finding the area of non-standard shapes is a bit different. You need to create regions *within* the shape for which you can find the area, and add these areas together.

Example

Find the area and perimeter of the polygon.



Solution:

- a. Finding the perimeter

To find the perimeter, add together the lengths of the sides. Start at the top and work clockwise around the shape.

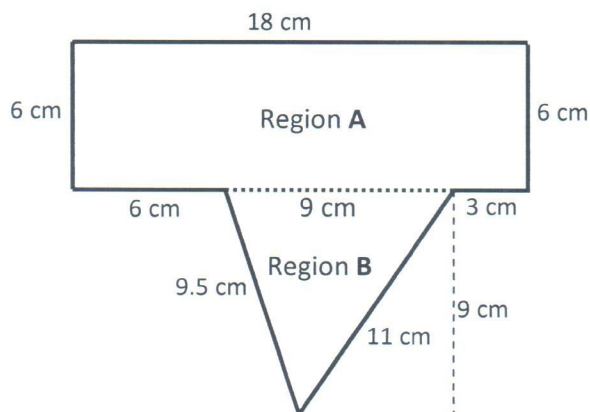
$$P = (18 + 6 + 3 + 11 + 9.5 + 6 + 6) \text{ cm}$$

$$P = 59.5 \text{ cm}$$



b. Finding the area

To find the area, divide the polygon into two separate, simpler regions. The area of the entire polygon will equal the sum of the areas of the two regions.



$$\text{Area of Polygon} = (\text{Area of A}) + (\text{Area of B})$$

Region A is a rectangle.

To find the area, multiply the length (18) by the width (6).

$$\begin{aligned}\text{Area of Region A} &= L \times W \\ &= (18 \text{ cm})(6 \text{ cm}) \\ &= 108 \text{ cm}^2\end{aligned}$$

Region B is a triangle.

To find the area, use the formula $A = \frac{1}{2}bh$, where the base is 9 cm and the height is 9 cm.

$$\begin{aligned}\text{Area of Region B} &= \frac{1}{2}bh \\ &= \frac{1}{2}(9 \text{ cm})(9 \text{ cm}) \\ &= \frac{1}{2}(81 \text{ cm}^2) \\ &= 40.5 \text{ cm}^2\end{aligned}$$

Substitute these two found areas in the formula:

$$\begin{aligned}\text{Area of Polygon} &= (\text{Area of A}) + (\text{Area of B}) \\ &= 108 \text{ cm}^2 + 40.5 \text{ cm}^2 \\ &= 148.5 \text{ cm}^2\end{aligned}$$

Therefore:

$$\text{Perimeter of the polygon} = 59.5 \text{ cm}$$

$$\text{Area of the polygon} = 148.5 \text{ cm}^2$$

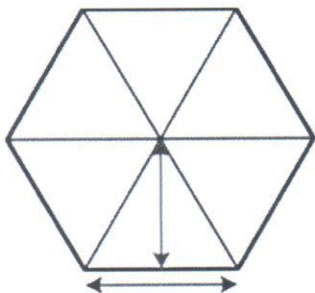


The area of any regular polygon

Regular polygons are polygons with all sides and all angles equal. To calculate the area of any regular polygon, the easiest way is to divide it into triangles, and use the formula for the area of a triangle.

Example 1

Find the area of the regular hexagon below whose side is 3 cm and height 4 cm.



Solution: Area = Area of triangle multiplied by the number of triangles

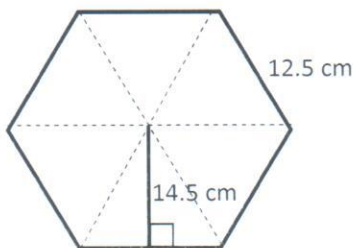
$$A = \frac{1}{2}(\text{base} \times \text{height}) \times 6 \text{ (because there are six triangles).}$$

$$A = \frac{1}{2}(3 \text{ cm})(4 \text{ cm})(6 \text{ cm})$$

$$A = 36 \text{ cm}^2$$

Example 2

Find the area of the regular polygon.



Solution:

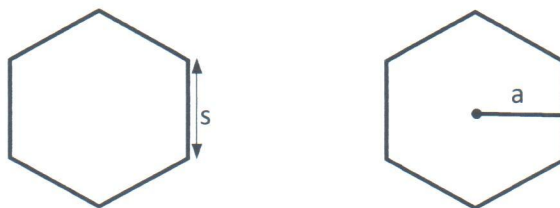
$$A = \frac{1}{2}b \times h \times \text{number of triangles}$$

$$A = \frac{1}{2}(12.5 \text{ cm})(14.5 \text{ cm})(6)$$

$$A = 543.75 \text{ cm}^2$$



Another way of calculating the area of regular polygon is using the formula that uses the perimeter and the shape's apothem.



The **perimeter** (p) is the combined length of the sides of the polygon. It is calculated by multiplying the length of one side (s) by the number of sides (n).

The **apothem** of a regular polygon is the shortest distance from the center point to one of the sides, creating a right angle. This is a little trickier to calculate than the perimeter.

The formula for calculating the length of the apothem (a) is this: the length of the side (s) divided by 2 times the tangent ($\tan$) of 180 degrees divided by the number of sides (n).

$$\text{Apothem (a)} = \frac{s}{2 \tan\left(\frac{180}{n}\right)}$$

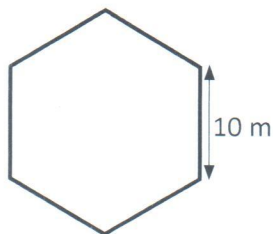
The **area** of any regular polygon is given by the formula:

$$\text{Area} = \frac{a \times p}{2}$$

where a is the length of the apothem and p is the perimeter of the polygon.

Example 3

Find the area bounded by the regular hexagon below if the sides are of length 10 metres as shown.



Solution: $n = 6$
 $s = 10$

- a. Finding for p : $p = n(s)$
 $= 6(10 \text{ m})$
 $= 60 \text{ m}$



b. Finding for the apothem:

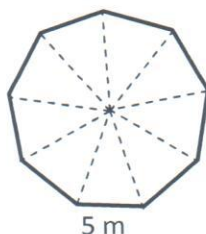
$$\begin{aligned}a &= \frac{s}{2 \tan\left(\frac{180}{n}\right)} \\&= \frac{10}{2 \tan\left(\frac{180}{6}\right)} \\&= \frac{10}{1.1547} \\&= 8.66 \text{ cm}\end{aligned}$$

c. Finding the area:

$$\begin{aligned}\text{Area} &= \frac{a \times p}{2} \\&= \frac{60(8.66)}{2} \\&= 259.8 \text{ cm}^2\end{aligned}$$

Example 4

Find the area bounded by the regular nonagon below if the sides are of length 5 metres as shown.



Solution: $n = 9$
 $s = 5$

a. to find for p:

$$\begin{aligned}p &= n(s) \\&= 9(5) \\&= 45 \text{ m}\end{aligned}$$

b. to find for the apothem:

$$\begin{aligned}a &= \frac{s}{2 \tan\left(\frac{180}{n}\right)} \\&= \frac{5}{2 \tan\left(\frac{180}{9}\right)} \\&= \frac{5}{0.728} \\&= 6.87 \text{ cm}\end{aligned}$$

d. to find the area:

$$\begin{aligned}\text{Area} &= \frac{a \times p}{2} \\&= \frac{45(6.87)}{2} \\&= 154.575 \text{ cm}^2\end{aligned}$$



Now do the learning activity.



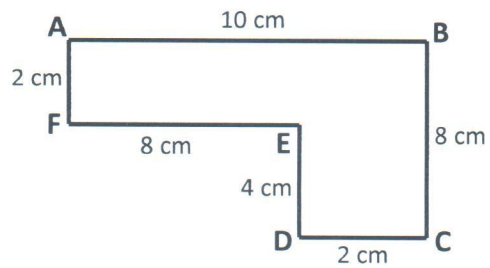
Learning Activity 12.1.2.4



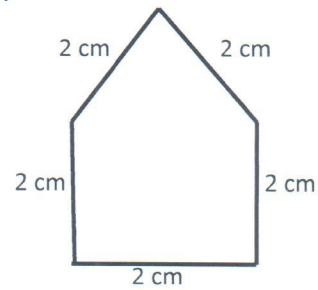
20 minutes

1. Find the perimeter of the given figure.

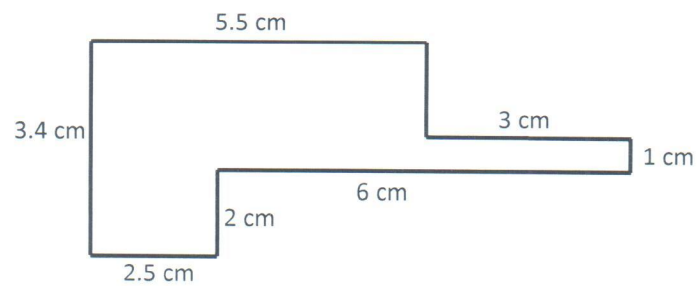
a.



b.

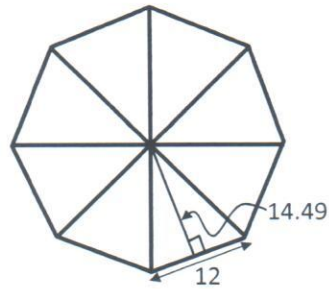


2. Find the area of the figure.





3. Find the area of the regular octagon below.



4. The width of a rectangle is 3 feet less than its length. The perimeter of the rectangle is 110 feet. Find its dimensions.
5. The height of a triangle is 4 cm more than twice the length of the base. The area of the triangle is 35 square cm. Find the height of the triangle.



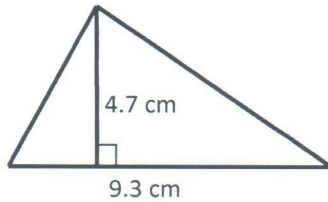
SUMMATIVE TASK 12.1.2



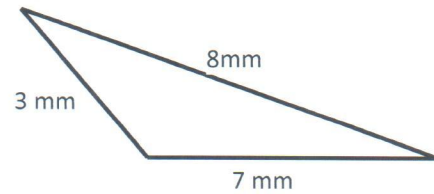
20 minutes

1. Find the area of each shape.

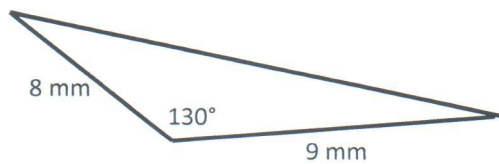
a.



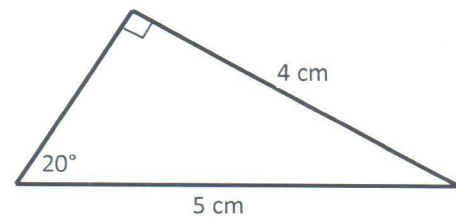
c.



b.

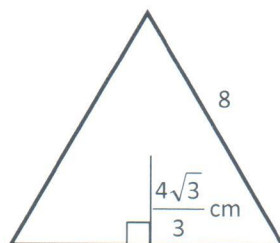


d.

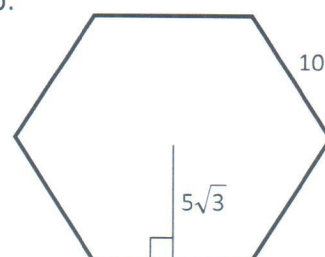


2. Find the area of each regular polygon. Leave your answer in surd form.

a.



b.





3. Find the perimeter of the triangle with sides: $a = 29$ mm $b = 38$ mm $c = 45$ mm

4. Find the area of triangle ABC with the given parts: $\angle A = 29^\circ$ $b = 15.2$ m $c = 6.5$ m

5. Find the area of triangle ABC with the given parts: $a = 46$ cm $b = 59$ cm $c = 69$ cm

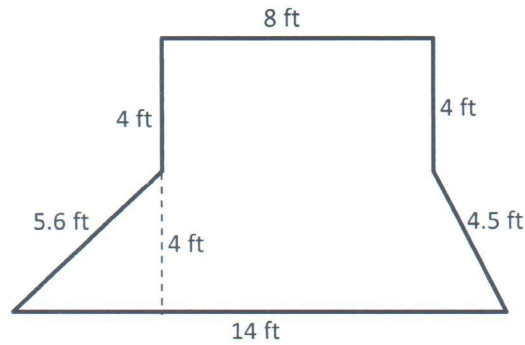
6. A painter needs to cover a triangular region 63 meters by 66 meters by 73 meters. A gallon of paint covers 70 square meters. How many gallons will be needed?

7. Find the area of the shape shown below.

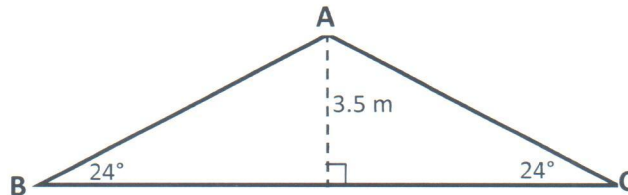




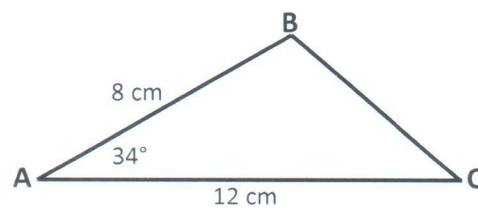
8. Rosie is planting a garden with the dimensions shown below. She wants to put a layer of organic matter on the entire surface of the garden. The materials cost K3 a square foot. How much money will she have to spend on these organic materials?



9. A roof is shaped like an isosceles triangle. The slope of the roof makes an angle of 24° with the horizontal, and has an altitude of 3.5 m. Determine the width of the roof to the nearest tenth of a metre.



10. Find the area of the triangle below.





2.1.3: SURFACE AREAS AND VOLUMES

In this topic, you will learn how to calculate the surface area and volume of common solids to include the definition of surface area, volume, prisms, cylinders, pyramids, cones, and spheres.

The **surface area** is the area that describes the material that will be used to cover a geometric solid. When we determine the surface areas of a geometric solid, we take the sum of the area for each geometric form within the solid.

The **volume** is a measure of how much a figure can hold and is measured in cubic units. The volume tells us something about the capacity of a figure.

Surface Area and Volume of Common Solids

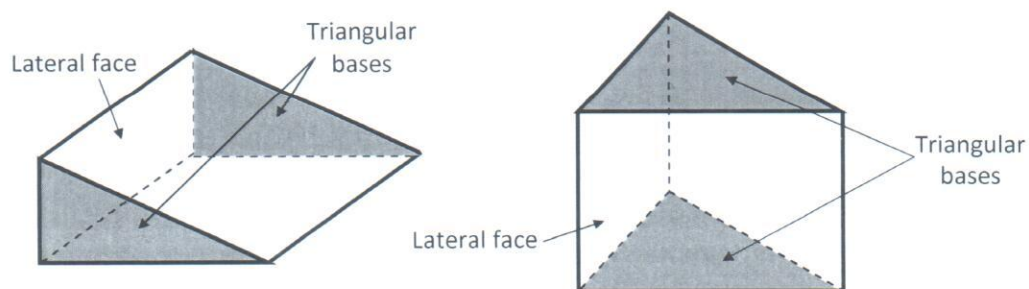
Prism

A **prism** is a solid that has two parallel faces which are congruent polygons at both ends. These faces form the **bases** of the prism. The other faces are in the shape of parallelograms. They are called **lateral faces**.

A prism is named after the shape of its base.

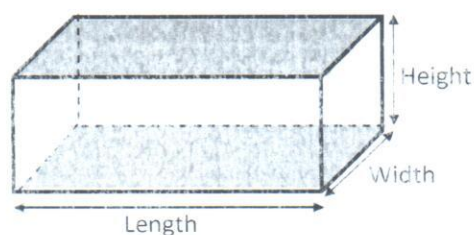
When the base of a prism is a triangle it is called **triangular prism**.

Below are examples of triangular prisms.



TRIANGULAR PRISM

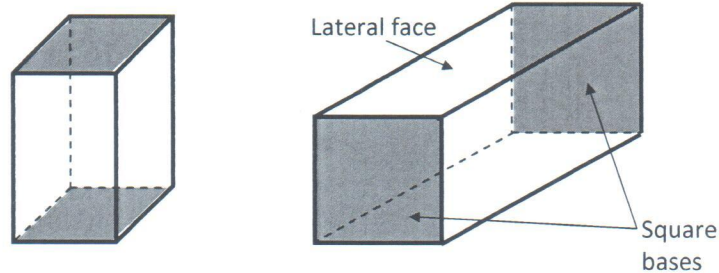
When all the six faces of a prism are rectangles it is called a **rectangular prism** or **cuboid**.



RECTANGULAR PRISM OR CUBOID

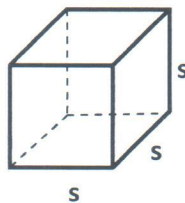


When the two bases of a prism are squares it is called a **square prism**.



SQUARE PRISM

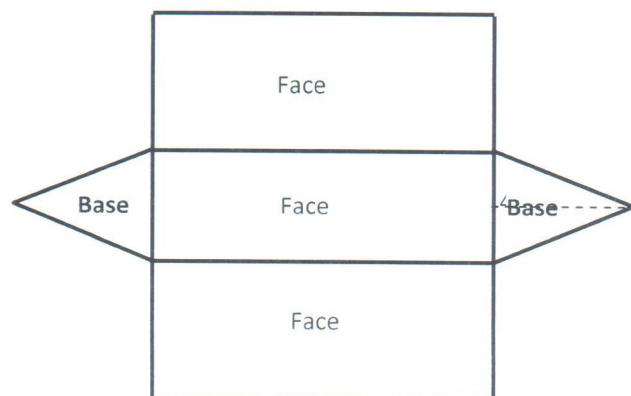
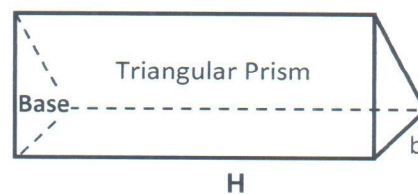
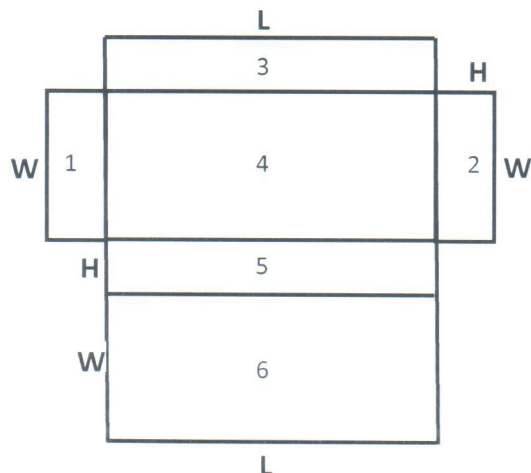
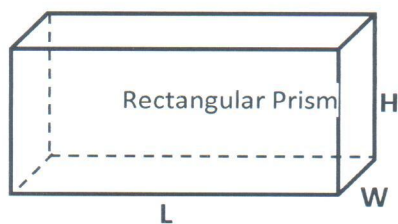
When all the three dimensions of a prism are equal and each six faces is a square, it is called a **cube**.

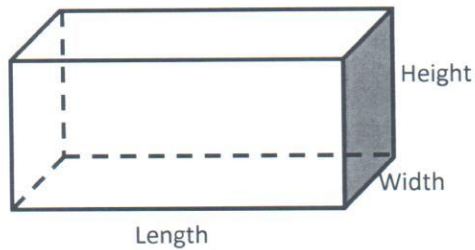


Finding the surface area of a prism

To find the surface area of a prism (or any other geometric solid), we open the solid like a carton box and flatten it out to find all included geometric forms. The pattern to which a prism or any 3D shape would look like if it will be opened out flat is called its **net**.

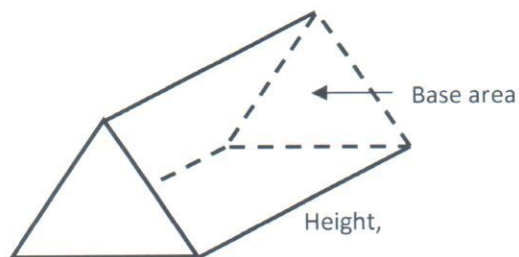
Below are prisms with their corresponding nets.



**RECTANGULAR PRISM**

$$V = L \times W \times H \text{ or } V = LWH$$

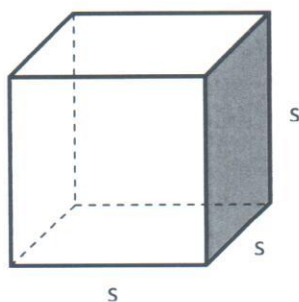
$$TSA = 2(LW + WH + LH)$$

**TRIANGULAR PRISM**

$$V = \left(\frac{1}{2}ab\right) \times h$$

$$TSA = 2(\text{Base area}) + 3(\text{Area of Lateral Face})$$

A cube is a three-dimensional figure with six matching square sides.

**CUBE**

$$V = s^3$$

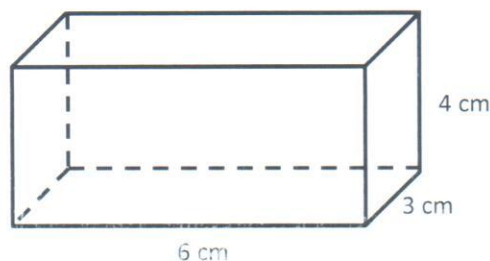
$$TSA = 6s^2$$

To find the volume of a prism (it does not matter if it is rectangular or triangular), we multiply the area of the base, called the base area B , by the height h .

$$V = Bh$$

Example 1

Find the surface area and volume of rectangular prism below.



Solution:

$$\text{Surface Area} = 2(LW + WH + LH)$$

$$= 2(6 \times 3 + 3 \times 4 + 6 \times 4)$$

$$= 2(18 + 12 + 24) = 108 \text{ cm}^2$$

$$\text{Volume} = L \times W \times H$$

$$= 6 \text{ cm} \times 3 \text{ cm} \times 4 \text{ cm}$$

$$= 72 \text{ cm}^3$$

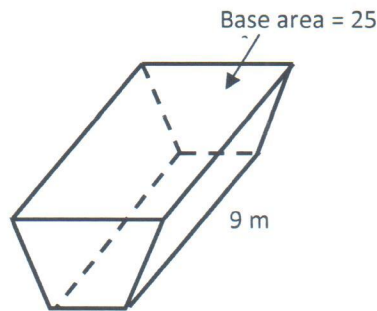


Example 2

Find the volume of the right prism.

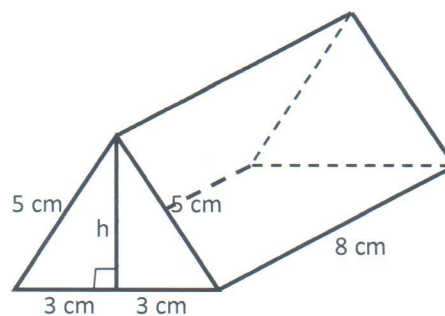
Solution:

$$\begin{aligned}\text{Volume} &= Bh \\ &= (25 \text{ cm}^2)(9 \text{ cm}) \\ &= 225 \text{ cm}^3\end{aligned}$$

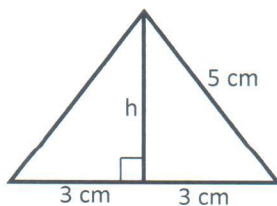


Example 3

Find the surface area and volume of the right prism.



Solution: First, we need to calculate the area of the triangular base.



We would need to use Pythagorean Theorem to calculate the height of the triangle.

$$\begin{aligned}h^2 + 3^2 &= 5^2 \\ h &= \sqrt{5^2 - 3^2} \\ h &= 4 \text{ cm}\end{aligned}$$

$$\text{Area of triangle} = \frac{1}{2}bh$$

$$\begin{aligned}&= \frac{1}{2} \times 6 \times 4 \\ &= 12 \text{ cm}^2\end{aligned}$$

$$\text{Area of rectangle} = L \times W$$

$$\begin{aligned}&= 6 \times 8 \\ &= 48 \text{ cm}^2\end{aligned}$$

$$\text{Surface Area} = 2 (\text{area of triangles}) + 3 (\text{area of rectangles})$$

$$= 2 (12) + 3 (48) = 168 \text{ cm}^2$$

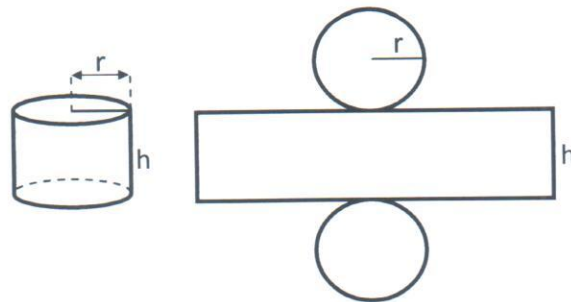
$$\text{Volume of prism} = Bh$$

$$\begin{aligned}\text{Volume of prism} &= 12 \text{ cm}^2 \times 8 \text{ cm} \\ &= 96 \text{ cm}^3\end{aligned}$$



Cylinder

A **cylinder** is a tube and is composed of two parallel congruent circles and a rectangle whose base is the circumference of the circle.



The surface area (**SA**) of a cylinder is twice the base area (**B**) plus the lateral area (**L**).

$$SA = 2B + L$$

The lateral area (**L**) is the base circumference $2\pi r$ times the height (**h**).

$$L = 2\pi rh$$

Each end (base) of the cylinder is a circle, with area πr^2 . Two ends are $2\pi r^2$.

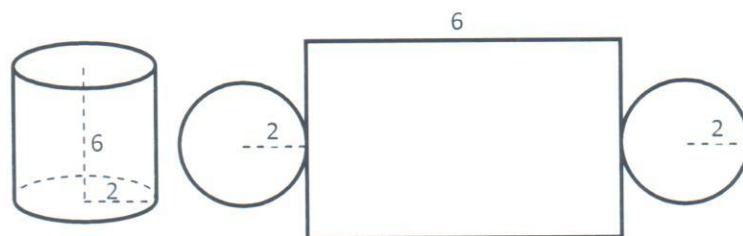
The side of the cylinder could be rolled out flat. It would then be a rectangle, length **L**. The edge where it joins the ends would be the same length as the circumference of the circular end, that is $2\pi r$.

So the area of the curved side is length x width = $2\pi rL$

So the total surface area of the cylinder is the two ends plus the side. $TSA = 2\pi r^2 + 2\pi rL$

This can be simplified to: $TSA = 2\pi r(r + L)$

Example



The area of one circle is:

$$\begin{aligned} A &= \pi r^2 \\ A &= \pi \times 2^2 \\ A &= 4 \times \pi \\ A &= 12.6 \end{aligned}$$

The circumference of a circle:

$$\begin{aligned} C &= \pi d \\ C &= \pi \times 4 \\ C &= 12.6 \end{aligned}$$

The area of the rectangle:

$$\begin{aligned} A &= C \times h \\ A &= 12.6 \times 6 \\ A &= 75.6 \end{aligned}$$

The surface area of the whole cylinder: $A = 75.6 + 12.6 + 12.6 = 100.8 \text{ units}^2$



To find the volume of a cylinder, multiply the base area (which is a circle) and the height, h .

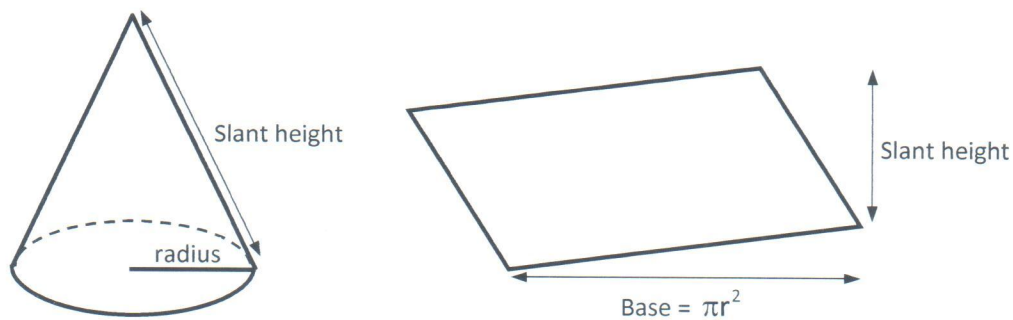
$$V = \pi r^2 \times h$$

$$V = 12.6 \times 6$$

$$V = 75.6 \text{ units}^3$$

Cone

The base of a cone is a circle and that is easy to see. The lateral surface of a cone is a parallelogram with a base that is half the circumference of the cone and with the slant height as the height. This can be a little bit trickier to see, but if you cut the lateral surface of the cone into sections and lay them next to each other it is easily seen.



The surface area S of a cone is the base area B plus the lateral surface area L .

$$S = B + L$$

Since the base of a right cone is a circle, $B = \pi r^2$ and the lateral area L is one-half the base circumference times the slant height (l), $L = \pi r l$.

Then, the surface area of a cone can also be written as: $S = \pi r^2 + \pi r l$ or $S = \pi r(r + l)$

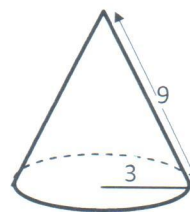
The volume of a cone is one third of the volume of a cylinder: $V = \frac{1}{3} \pi r^2 h$

Example

Find the surface area and the volume of the cone below. Use $\pi = 3.14$

Solution:

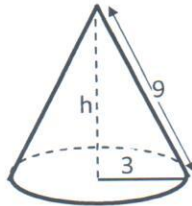
$$\begin{aligned} \text{a) Surface area: } SA &= \pi r^2 + \pi r l \\ &= 3.14(3)^2 + 3.14(3)(9) \\ &= 3.14(9) + 3.14(27) \\ &= 28.26 + 84.78 \\ &= 113.04 \text{ square units} \end{aligned}$$





b) Volume: $V = \frac{1}{3}\pi r^2 h$

As you can see in this case, the height (**h**) is not given, so to find the volume, we have to find the height first using the Pythagorean Theorem.



$$c^2 = a^2 + b^2$$

$$9^2 = 3^2 + b^2$$

$$81 = 9 + b^2$$

$$b^2 = 81 - 9$$

$$b^2 = 72$$

$$b = 8.5 \text{ units}$$

where **b** is the height of the cone

Addition Property of Equality

Get the square root of both sides

Answer correct to 1 decimal place

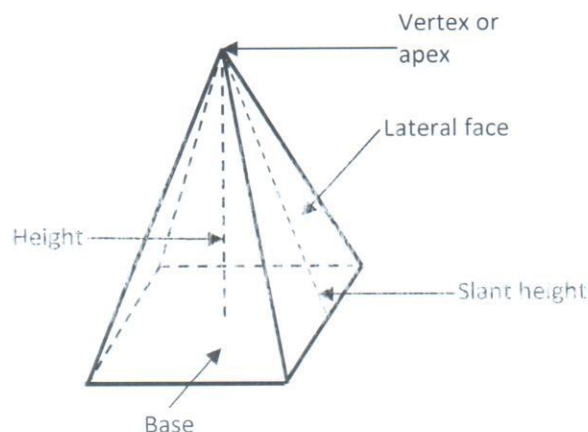
Now we find the volume of the cone.

$$\begin{aligned}\text{Volume} &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}(3.14)(3)^2(8.5) \\ &= \frac{1}{3}(3.14)(9)(8.5) \\ &= \frac{1}{3}(240.21) \\ &= 80.07 \text{ cubic units}\end{aligned}$$

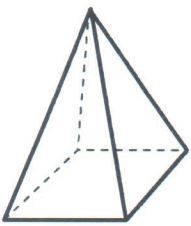
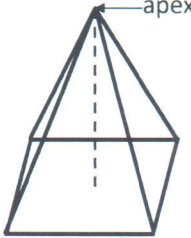
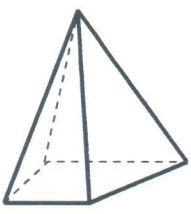
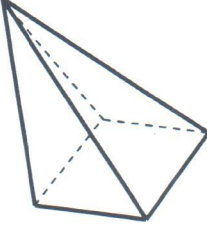
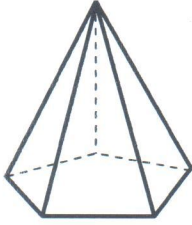
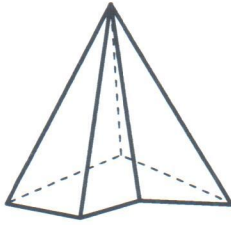
12.1.3.1: Surface Areas of Pyramids, Spheres and Truncated Solids

Surface Areas of Pyramids

A pyramid is a solid with a polygonal **base** and several triangular **lateral faces**. The lateral faces meet at a common **vertex**. The number of lateral faces depends on the number of sides of the base. The **height** of the pyramid is the perpendicular distance from the base to the vertex.

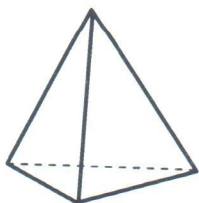


**Types of Pyramids**

Regular Pyramid  The base of a regular pyramid is a regular polygon and its faces are equally sized.	Right Pyramid  A right pyramid has isosceles triangles as its lateral faces and its apex lies directly above the midpoint of the base.
Irregular Pyramid  The base of an irregular pyramid is an irregular polygon, and as a result, its faces are not equally sized.	Oblique Pyramid  An oblique pyramid does not have all isosceles triangles as its sides.
Convex Pyramid  A convex pyramid has a convex polygon as its base.	Concave Pyramid  A concave pyramid has a concave polygon as its base.

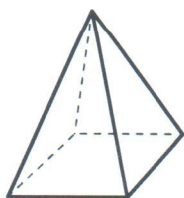
Types of the Pyramids by their Base

Triangular Pyramid



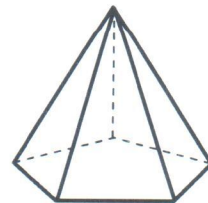
The base is a triangle.

Square Pyramid



The base is a square.

Pentagonal Pyramid



The base is a pentagon.

We can find the surface area of any pyramid by adding up the areas of its lateral faces and its base.

Surface area of any pyramid = area of base + area of each of the lateral faces

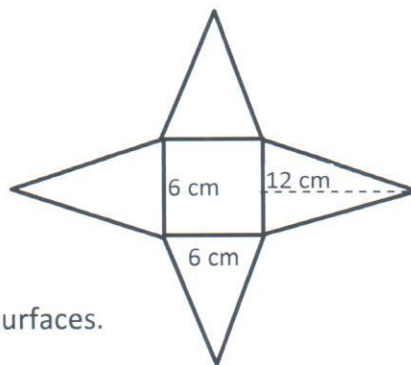
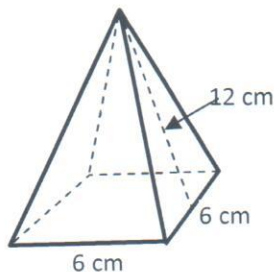
Surface area of regular pyramid = area of base + $\frac{1}{2}ps$, where p is the perimeter of the base and s is the slant height.



Surface area of square pyramid = $b^2 + 2bs$, where b is the length of the base and s is the slant height.

Example 1

Calculate the surface area of the following pyramid.



Solution:

Sketch a net of the above pyramid to visualize the surfaces.

Since the given pyramid is a square pyramid, we can use any of the formulas.

Solution 1 Using the formula for the surface area of any pyramid

$$\text{Area of base} = (6)(6) = 36 \text{ cm}^2$$

$$\text{Area of the four triangles} = \frac{1}{2}(6)(12)(4) = 144 \text{ cm}^2$$

$$\begin{aligned}\text{Total surface area} &= \text{area of the base} + \text{area of triangles} \\ &= 36 + 144 = 180 \text{ cm}^2\end{aligned}$$

Solution 2 Using the formula for a regular pyramid:

$$\begin{aligned}\text{Surface area of regular pyramid} &= \text{area of base} + \frac{1}{2}ps \\ &= (6)(6) + \frac{1}{2}(6)(4)(12) \\ &= 180 \text{ cm}^2\end{aligned}$$

Solution 3 Using the formula for a square pyramid

$$\begin{aligned}\text{Surface area of square pyramid} &= b^2 + 2bs \\ &= (6)(6) + (2)(6)(12) \\ &= 180 \text{ cm}^2\end{aligned}$$

**Example 2**

Find the surface area of pyramid whose side faces are the same, slant height is 3, and the base is a square with side length of 5.

Solution: Area of base = 5^2 ; perimeter of the base = $4(5)$; slant height (S) = 3

$$\text{Surface area} = \text{area of base} + \frac{1}{2}ps$$

$$= 5^2 + \frac{1}{2}(4)(5)(3)$$

$$= 25 + \frac{1}{2}(60)$$

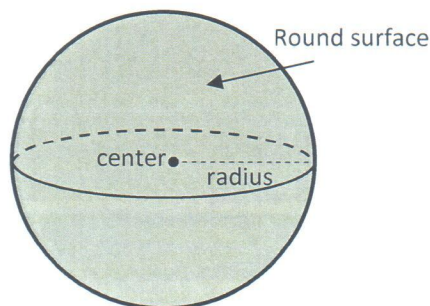
$$= 25 + 30$$

$$= 55$$

The surface area is 55 square units.

Surface Area of a Sphere

A **sphere** is a solid in which all the points on the round surface are equidistant from a **fixed point**, known as the centre of the sphere. The distance from the centre to the surface is the **radius**.



The **center** is the interior point that is equidistant to all points on the surface of the sphere.

The distance from the center to any point on the surface of the sphere is called **radius**.

The **diameter** is the distance from one point through the centre to another point.

$$\text{Surface Area of sphere} = 4\pi r^2, \text{ where } r \text{ is the radius.}$$



Example Calculate the surface area of a sphere with radius 3.2 cm.

Solution:

Surface area of sphere = $4\pi r^2$

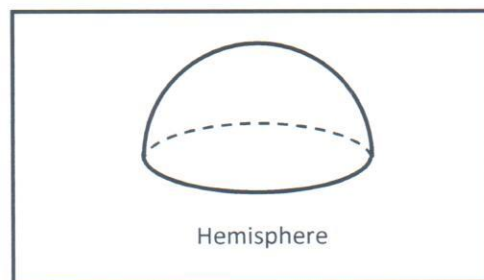
$$SA = 4\pi (3.2)^2$$

$$SA = 4(3.14)(10.24)$$

$$SA = 128.6 \text{ cm}^2$$

Hemisphere

A **hemisphere** is half a sphere, with one flat circular face and one bowl-shaped face.



The surface area of a hemisphere is equal to the area of the curve surface plus the area of the circular base.

Surface area of hemisphere = $\frac{1}{2} \times$ surface area of sphere + area of base

$$SA = \frac{1}{2} 4\pi r^2 + \pi r^2 = 3\pi r^2$$

Example Calculate the surface area of a hemisphere with radius 4 cm.

Solution: Surface area of hemisphere = $\frac{1}{2} \times$ surface area of sphere + area of base

$$= \frac{1}{2} (4\pi r^2 + \pi r^2)$$

$$= 3\pi r^2$$

$$= 3\pi (4)^2$$

$$= 150.86 \text{ cm}^2$$

Now do the learning activity.

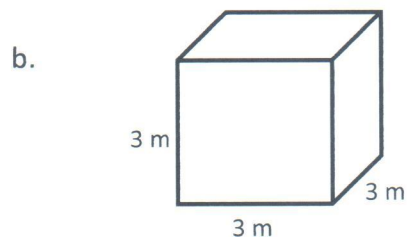


Learning Activity 12.1.3.1



20 minutes

1. Find the surface area of the solids below.



- c. A cone with diameter 10 cm and a slant height of 13 cm.
- d. A soft drink can has a diameter of 6 cm and a height of 11.5 cm.

2. Find the surface area of a sphere with a diameter of 20 cm.

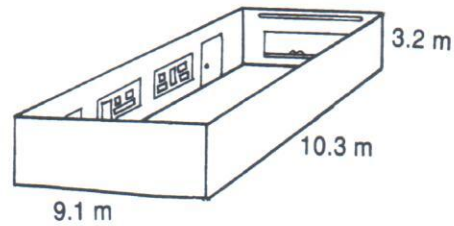
3. Find the surface area of a square pyramid measuring 9 cm along the base with a slant height of 12.8 cm.



4. A classroom has the dimensions shown.

Find:

- the area of the floor.
- the total area of the four walls.



5. A paper coffee filter has a radius of 6 cm and a slant height of 12 cm. Find the surface area of the filter. Round off your answer to the nearest whole number.

12.1.3.2: Volume of Pyramids, Spheres and Truncated Solids

Volume of Pyramids

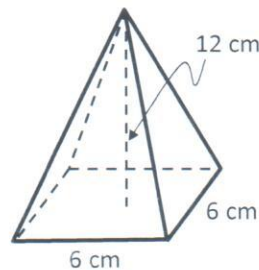
The Volume of a pyramid is given by the formula:

$$V = \frac{1}{3} Ah$$

where **A** is the area of the base and **h** is the height of the pyramid.

Example 1

Calculate the volume of the pyramid shown.



Solution:

To calculate the volume, we find the area of the base which is $6 \times 6 = 36 \text{ cm}^2$; height is 12 cm.

Substituting in the formula, Volume = $\frac{1}{3} Ah$

$$\begin{aligned} &= \frac{1}{3} (36 \text{ cm})(12 \text{ cm}) \\ &= 144 \text{ cm}^3 \end{aligned}$$



Example 2

The base of a pyramid is a square with side length of 5, height is 6. Find the volume.

Solution:

$$\begin{aligned}\text{Volume} &= \frac{1}{3} Ah \\ &= \frac{1}{3} (5)(5)(6) \\ &= \frac{1}{3} (150) \\ &= 50\end{aligned}$$

The volume is 50 cubic units.

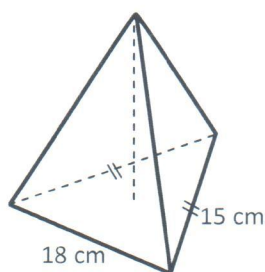
Example 3

Find the volume of a pyramid with a rectangular base measuring 6 cm by 4 cm and a height 10 cm.

$$\begin{aligned}\text{Solution:} \quad \text{Volume} &= \frac{1}{3} Ah \\ &= \frac{1}{3} (6\text{cm})(4\text{ cm})(10\text{ cm}) \\ &= 80\text{ cm}^3\end{aligned}$$

Example 4

The following figure is a right pyramid with an isosceles triangle base. Find the volume of the pyramid if the height is 20 cm.



Solution:

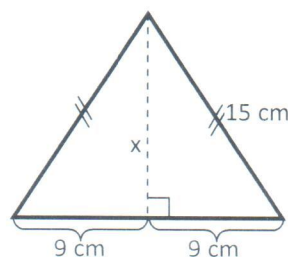
First, we have to calculate the area of the base. To do that, we would need to get the height of the isosceles triangle that forms the base.

Using Pythagorean Theorem: $x = \sqrt{15^2 - 9^2}$

$$x = \sqrt{225 - 81}$$

$$x = \sqrt{144}$$

$$x = 12\text{ cm}$$





Now find the area of the isosceles triangle base using the formula: $A = \frac{1}{2} ab$

$$A = \frac{1}{2} (18 \text{ cm})(12 \text{ cm})$$

$$= \frac{1}{2} (216 \text{ cm}^2)$$

$$= 108 \text{ cm}^2$$

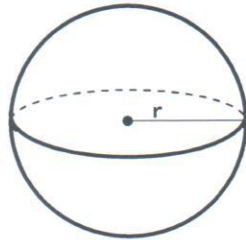
Hence, Volume of pyramid, we have $V = \frac{1}{3} Ah$

$$= \frac{1}{3} (108 \text{ cm}^2)(20 \text{ cm})$$

$$= \frac{1}{3} (2160 \text{ cm}^3)$$

$$= 720 \text{ cm}^3$$

Volume of a Sphere



The Volume of a sphere is given by the formula:

$$V = \frac{4}{3} \pi r^3$$

where r is the radius of the sphere.

Example 1

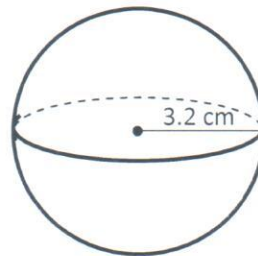
Calculate the volume of a sphere with radius 3.2 cm.

Solution: Using the formula $V = \frac{4}{3} \pi r^3$

$$V = \frac{4}{3} \pi (3.2)^3$$

$$V = \frac{4}{3} (3.14) (3.2)(3.2)(3.2)$$

$$V = 137.2 \text{ cm}^3$$



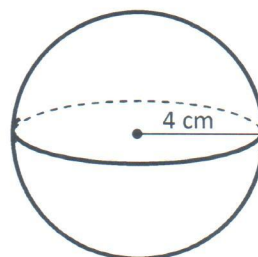


Example 2

Calculate the volume of sphere with radius 4 cm. Use $\pi = \frac{22}{7}$.

Solution: Using the formula $V = \frac{4}{3}\pi r^3$

$$V = \frac{4}{3}\left(\frac{22}{7}\right)(4^3)$$
$$V = \frac{4}{3}\left(\frac{22}{7}\right)(64)$$
$$V = 268.19 \text{ cm}^3$$

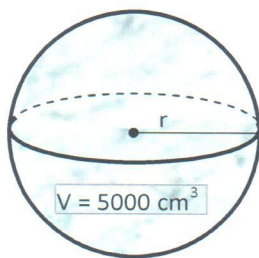


We can also **change the subject of the formula** to obtain the radius given the volume.

From the formula $V = \frac{4}{3}\pi r^3$, we can derive the formula for the radius by making r as the subject of the formula. Hence, $r = \sqrt[3]{\frac{3V}{4\pi}}$.

Example 3

The volume of a spherical ball is $5,000 \text{ cm}^3$. What is the radius of the ball?



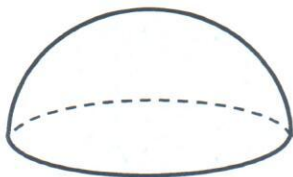
Solution: Using the formula $r = \sqrt[3]{\frac{3V}{4\pi}}$

$$r = \sqrt[3]{\frac{3(5000 \text{ cm}^3)}{4(3.14)}}$$
$$r = \sqrt[3]{\frac{15000 \text{ cm}^3}{12.56}}$$
$$r = \sqrt[3]{1194.28 \text{ cm}^3}$$
$$r = 10.61 \text{ cm}$$



Volume of a Hemisphere

The volume of a hemisphere is equal to one- half of the volume of a sphere.



$$V = \frac{1}{2} \left(\frac{4}{3} \pi r^3 \right)$$

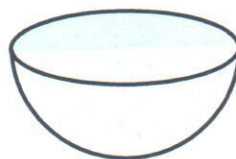
Example 1

The volume of a hemispherical bowl is 2425.5 cm^3 . Find its radius and surface area.

Solution:

Let the radius be $r \text{ cm}$.

$$V = 2425.5 \text{ cm}^3$$



$$V = \frac{1}{2} \left(\frac{4}{3} \pi r^3 \right)$$

$$2425.5 = \frac{1}{2} \left(\frac{4}{3} \pi r^3 \right) \quad \text{substitute volume}$$

$$2425.5 \times 6 = 4\pi r^3 \quad \text{multiply both sides by 6}$$

$$14\,553 = 4\pi r^3$$

$$\frac{14553}{4\pi} = r^3 \quad \text{divide both sides by } 4\pi$$

$$1158.68 = r^3 \quad \text{get cube root of both sides}$$

$$\sqrt[3]{1158.68} = r$$

$$r = 10.5 \text{ cm.}$$

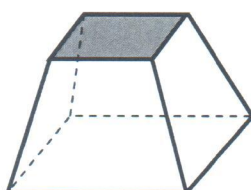
Therefore the radius of the bowl is **10.5 cm**.



Truncated Solids

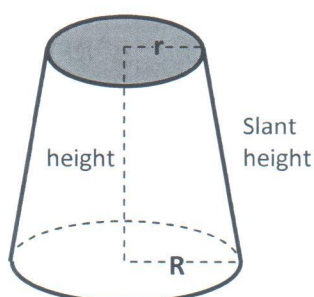
When a solid is cut off by a plane parallel to the base, the part that remains is called a **frustum** or a **truncated solid**.

The figures shown below are examples of truncated solids.



We consider truncated figures as being cut off by a plane surface.

A **truncated cone** is the result of cutting a **cone** by a plane parallel to the base and removing the part containing the apex.

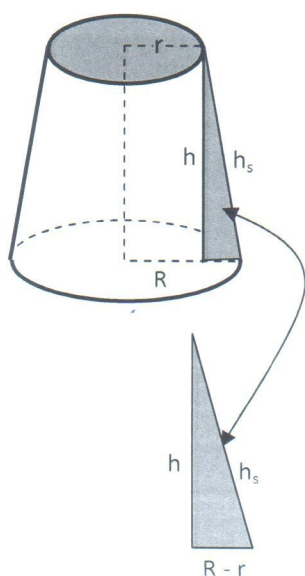


The height (**h**) is the line segment that joins the two bases perpendicularly.

The radii are the radii of their bases.

The slant height (**h_s**) is the shortest possible distance between the edges of the two bases.

Calculation of the Apothem of a Truncated Cone



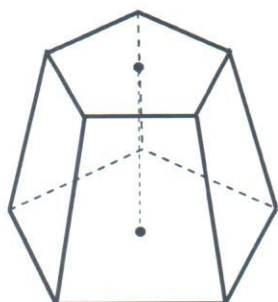
The slant height (**h_s**) of the truncated cone is obtained by applying the Pythagorean theorem for the shaded triangle.

$$h_s^2 = h^2 + (R-r)^2$$

$$h_s = \sqrt{h^2 + (R-r)^2}$$



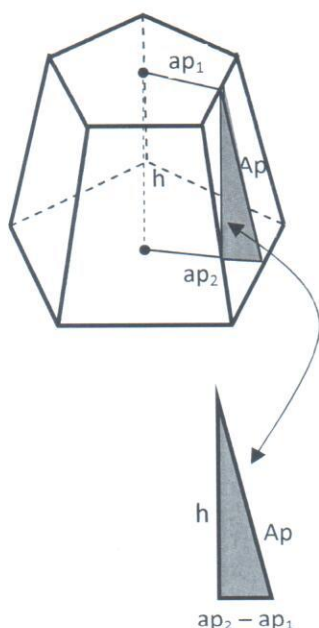
A **truncated pyramid** is the result of cutting a pyramid by a plane parallel to the base and separating the part containing the apex.



The lateral faces of a truncated pyramid are trapezoids.

The height of a truncated pyramid is the perpendicular distance between the bases.

Calculation of the Apothem of a Truncated Pyramid



To calculate the apothem of a truncated pyramid, the height, the apothem of the biggest base and the apothem of the minor base must be known. Apply the Pythagorean theorem to determine the length of the hypotenuse of the shaded triangle to obtain the apothem.

$$Ap^2 = h^2 + (ap_2 - ap_1)^2$$

The lateral area of the frustum of a right circular cone and a regular pyramid

The formula for the lateral area of the frustum of a right circular cone is given as:

$$LA = \pi h_s (R + r)$$

Where,
 LA = lateral area
 h_s = slant height
 R = radius of larger base
 r = radius of smaller base

The lateral area of the frustum of a regular pyramid is given as:

$$LA = \frac{1}{2} h_s (P_B + P_b)$$

Where,
 LA = lateral area
 h_s = slant height
 P_B = perimeter of larger base
 P_b = perimeter of smaller base



The Surface Area of the frustum of a pyramid or cone

The surface area of the frustum of a pyramid or cone equals the sum of the lateral area, the larger base area, and the smaller base area

$$\text{TSA} = \text{LA} + A_B + A_b$$

Where, TSA = surface area

LA = lateral area

A_B = area of larger base

A_b = area of smaller base

The volume of the frustum of a pyramid or cone

The volume of the frustum of a pyramid is given as:

$$V = \frac{1}{3}h(A_B + A_b + \sqrt{A_B A_b})$$

Where, V = volume

h = height

A_B = area of the larger base

A_b = area of the smaller base

The formula for the volume of a frustum of a right circular cone is given as:

$$V = \frac{1}{3}\pi h(R^2 + r^2 + Rr)$$

Where, V = volume

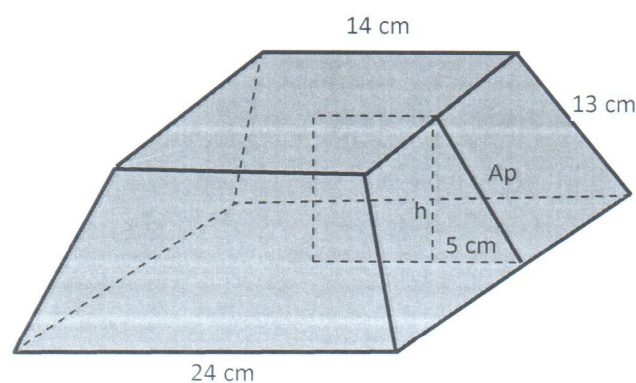
h = height

R = radius of larger base

r = radius of smaller base

Example 1

Calculate the lateral area, surface area and volume of the truncated square pyramid whose larger base edge is 24 cm, smaller base edge is 14 cm and whose lateral edge is 13 cm.





Solution:

- a. To find the lateral area we use the formula: $LA = \frac{1}{2} h_s(P_B + P_b)$

First, use Pythagorean Theorem to find h. $13^2 = h^2 + 5^2$
 $h = \sqrt{13^2 - 5^2}$
 $= 12 \text{ cm}$

Find perimeter of the larger base: $P_B = 4(24 \text{ cm})$
 $= 96 \text{ cm}$

Find perimeter of the smaller base: $P_b = 4(14 \text{ cm})$
 $= 56 \text{ cm}$

Substitute these results in the formula: $LA = \frac{1}{2} h_s(P_B + P_b)$
 $= \frac{1}{2} (12)(96 + 56)$
 $= \frac{1}{2} (12)(152)$
 $= 912 \text{ cm}^2$

- b. To find the Total Surface Area use the formula: $TSA = LA + A_B + A_b$

Find area of larger base: $A_B = s^2 = 24^2 = 576 \text{ cm}^2$

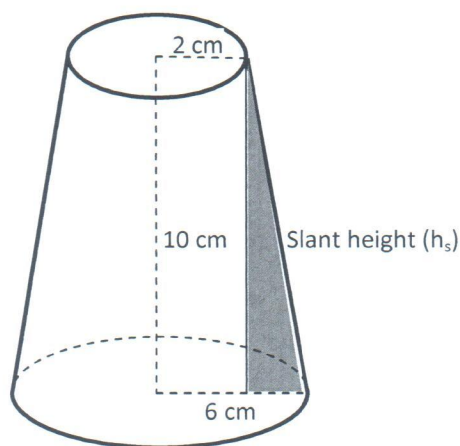
Find area of smaller base: $A_b = s^2 = 14^2 = 196 \text{ cm}^2$

Substitute these results to the formula: $TSA = LA + A_B + A_b$
 $= 912 \text{ cm}^2 + 576 \text{ cm}^2 + 196 \text{ cm}^2$
 $= 1684 \text{ cm}^2$

- c. To find the volume use the formula: $V = \frac{1}{3} h(A_B + A_b + \sqrt{A_B A_b})$
 $V = \frac{1}{3} [12](576 + 196 + \sqrt{(576)(196)})$
 $V = \frac{1}{3} (12)(772 + \sqrt{112896})$
 $V = \frac{1}{3} (12)(772 + 336)$
 $V = 4(1108)$
 $V = 4432 \text{ cm}^3$



2. Calculate the lateral area, surface area and volume of a truncated cone of radii 2 and 6 cm and a height of 10 cm.



Solution:

- a. To find the lateral area, first find the slant height (h_s) using Pythagorean Theorem.

$$\begin{aligned}h_s^2 &= 10^2 + (6 - 2)^2 \\&= \sqrt{10^2 + (6 - 2)^2} \\&= \sqrt{100 + 16} \\&= \sqrt{116} \\&= 10.77 \text{ cm}\end{aligned}$$

So, Lateral area of the truncated cone is $LA = \pi h_s (R + r)$

$$\begin{aligned}&= 3.14(10.77)(6 + 2) \\&= (33.8178)(8) \\&= 270.68 \text{ cm}^2\end{aligned}$$

- b. To find surface area use the formula: $TSA = LA + A_B + A_b$

$$\begin{aligned}&= 270.68 + (3.14)(6^2) + (3.14)(2^2) \\&= 270.68 + 113.04 + 12.56 \\&= 396.28 \text{ cm}^2\end{aligned}$$

- c. To find volume, use the formula: $V = \frac{1}{3} \pi h (R^2 + r^2 + Rr)$

$$\begin{aligned}V &= \frac{1}{3} (3.14)(10)[(6^2 + 2^2 + (6)(2))] \\&= 544.27 \text{ cm}^3\end{aligned}$$



Here is a summary of the most common solids.

Shape	Surface Area Formula	Volume Formula
Cube	$6s^2$ where s = length of the side	$V = s^3$ where s = length of the side
Cuboid	$2(lw + lh + wh)$ where l = length, w = width, h = height	$V = LWH$ where l = length, w = width, h = height
Prism	$2 \times \text{area of base} + \text{perimeter of base} \times \text{height}$	$\text{area of base} \times \text{height}$
Cylinder	$2\pi r(r + h)$ where r = radius, h = height	$V = \pi r^2 h$ where r = radius, h = height
Hollow Cylinder	$2\pi rh + 2\pi Rh + 2(\pi R^2 - \pi r^2)$ where R = radius of the outer surface, r = radius of the inner surface	$V = \pi h(R^2 - r^2)$ where R = radius of the outer surface, r = radius of the inner surface
Cone	$\pi r(r + l)$ where r = radius, s = slant height	$V = \frac{1}{3} \pi r^2 h$ where r = radius, h = height
Pyramid	Any pyramid = area of base + area of each of the lateral faces Regular pyramid = area of base + $\frac{1}{2} ps$ where p = perimeter of the base, s = slant height Square pyramid = $b^2 + 2bs$ where b = length of the base, s = slant height	$V = \frac{1}{3} Bh$ where B = area of base h = height of the pyramid
Sphere	$4\pi r^2$ where r = radius	$V = \frac{4}{3} \pi r^3$ where r = radius
Hemisphere	$3\pi r^2$ where r = radius	$V = \frac{2}{3} \pi r^3$ where r = radius

Now do the learning activity.

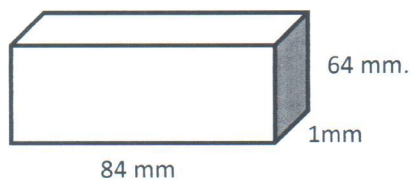


Learning Activity 12.1.3.2



20 minutes

1. Find the volume of the box.



2. Calculate the volume of a cylinder where:

a) the area of the base is 20 cm^2 and the height is 6 cm.

b) the radius of the base is 10 cm and the height is 15 cm.

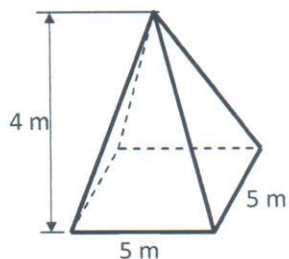
3. Calculate the volume of a cone if the height is 12 cm and the radius is 7 cm.

4. Find the volume of a pyramid with a rectangular base measuring 6 cm by 4 cm and height 10 cm.

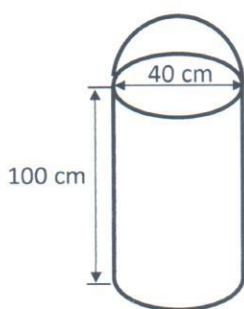
5. A ball has a volume of 123 cm^3 . Find its radius to the nearest cm.



6. Find the volume of a square pyramid with the given side 5 m and vertical height 4 m.



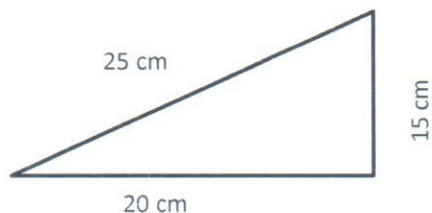
7. Calculate the volume of a water cylinder with total height 100 cm, diameter 40 cm and whose top section is a hemisphere.

**SUMMATIVE TASK 12.1.3**

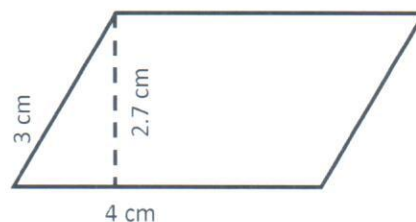
20 minutes

1. Find the perimeter and area of the figures below.

a) Perimeter = _____
Area = _____



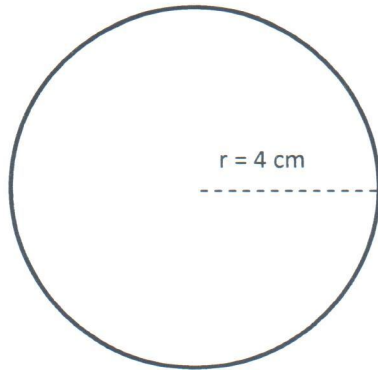
b) Perimeter = _____
Area = _____



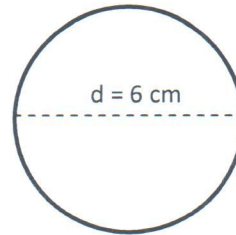


2. Find the area of the figures below.

a) Area = _____



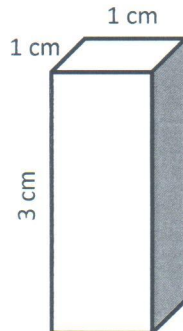
b) Area = _____



3. Find the volume and surface area of the figures below.

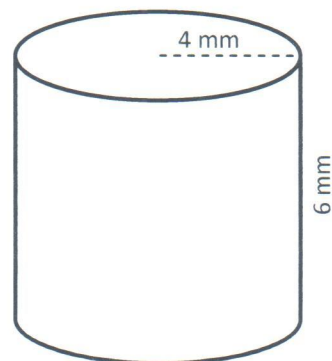
a) Volume = _____

Surface Area = _____

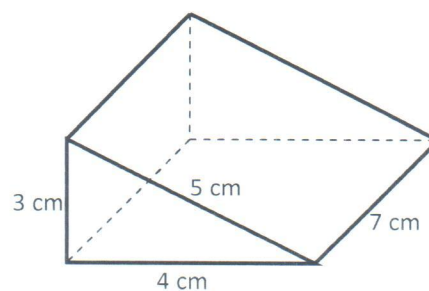


b) Volume = _____

Surface Area = _____

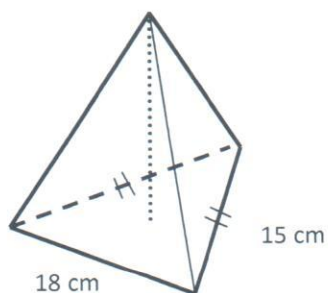


4. Calculate the surface area of the following prism.

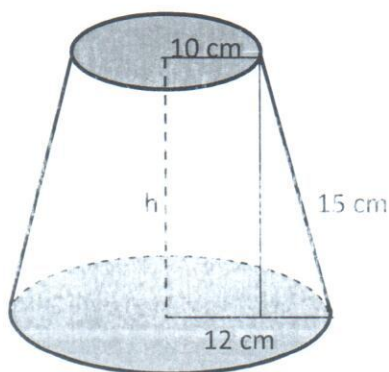




5. Calculate the volume of a cylinder where:
- the area of the base is 30 cm^2 and the height is 6 cm.
 - the radius of the base is 14 cm and the height is 10 cm.
6. A cone has a circular base of radius 10 cm and a slant height of 30 cm. Calculate the surface area.
7. The volume of a spherical ball is $5\,000 \text{ cm}^3$. What is the radius of the ball?
8. The following figure is a right pyramid with an isosceles triangle base. Find the volume of the pyramid if the height is 20 cm.



9. Calculate the lateral area, surface area and volume of a truncated cone of radii 10 and 12 cm and a slant height of 15 cm.





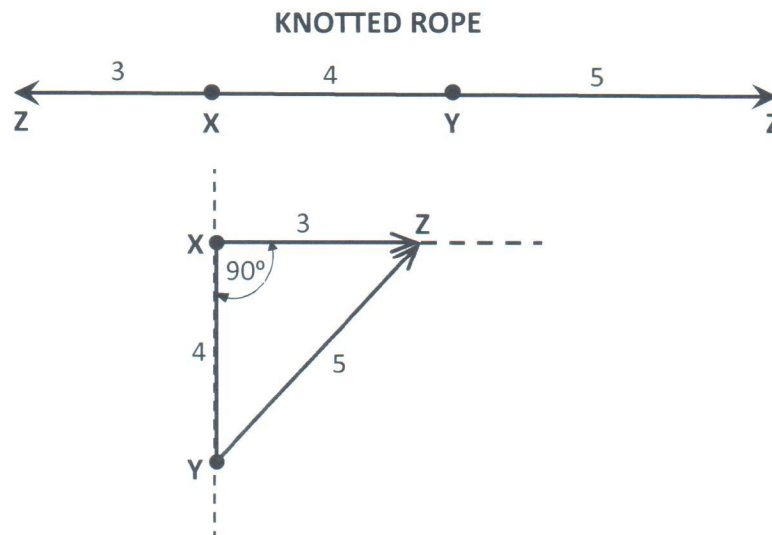
12.1.4: SURVEYING

Surveying involves the measurement of length over the ground using some unit of length such as a pace, the circumference of a trundle wheel or the distance between the ends of a tape or chain. Land surveys measure an area of land and show it using a suitable scale on a map or plan.

History

Surveying is the science and art of measuring distances and angles on or near the surface of the earth. It is an orderly process of acquiring data relating to the physical characteristics of the earth and in particular the relative position of points and the magnitude of areas. Evidence of surveying and recorded information exists as early as five thousand years in China, India, Babylon and Egypt.

Ancient Egyptian surveyors were called **harpedonapata** (rope-stretcher). They used ropes and knots, tied at pre-determined intervals, to measure distances. The 3-4-5 triangle (later formalised by Pythagoras) was discovered to give a right angle easily by using a rope knotted at distances of 3, 4 and 5 units (See below) and shaped (stretched) to form a triangle with a knot at each corner (vertex).



Land surveying is one of the world's oldest professions and dates back as far as the ancient Egyptians. In this topic we shall look at land survey techniques, and how they are used.

History of Land Survey Techniques

As we mentioned in our introduction, land surveying dates back thousands of years and is an integral part of every major civilization. Ever since the beginning of civilization, man needs to know what property is his and understand the shape of the land.



The basics fundamentals of land surveying have not changed since it first began; however, the land survey equipment has changed drastically over the years.

In the past, land survey techniques had the right principles but due to less knowledge and technology, they were not accurate or reliable. Modern land survey techniques are incredibly precise largely due to advanced technology available to create the tools used to perform the land survey techniques.

Essentially land survey techniques have remained very much the same – using fundamentals of mathematics including geometry, calculus and trigonometry. It is the tools that have changed.

12.1.4.1: Surveying on Level Grounds without Obstacles

It is essential that the size and shape of the Earth's surface be accurately recorded. The task of the surveyor is to link the shapes and sizes of parts of our real three-dimensional world to the two-dimensional survey map drawn to scale. First, we shall investigate working on level ground without obstacles, then later we shall investigate more difficult surveys on level ground with obstacles.

Chaining (Linear Surveying)

Chaining involves producing a plan of a limited area using simple distance measuring instruments. It is most appropriate for areas that are relatively free of obstructions or level grounds without obstacles and not too rough. In this form of surveying, distances can be measured by using pacing, tapes or chains, trundle wheels or odometer devices or electronic devices.

Pacing

Chain surveying uses only the accurate measurement of length. One of the earliest forms of measuring length is **pacing**. It is not highly accurate but it is useful because it does not rely on any equipment except your body. The accuracy of the results of surveying using pacing can be improved if you have a good idea of the length of your paces.

Pace length is calculated by:

$$\text{Pace length} = \frac{\text{Number of metres}}{\text{Number of paces}}$$

Once the length of your pace is known, you can estimate various lengths with modern accuracy. In this way, the areas of various shapes can also be calculated.



Estimating Distance and Area using Pace Length

Example 1

A person who takes 25 paces to cover 20 m finds that a square field has a side that is 370 paces long. Calculate the side and the area of the field.

Solution:

$$\begin{aligned}\text{Pace length} &= \frac{\text{Number of metres}}{\text{Number of paces}} \\ &= \frac{20 \text{ m}}{25 \text{ paces}} \\ &= 0.8 \text{ m per pace}\end{aligned}$$

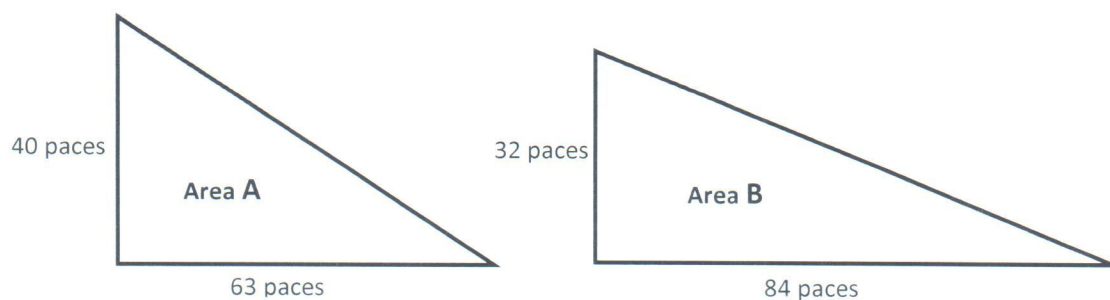
$$\begin{aligned}\text{Length of field} &= 370 \text{ paces} \\ &= 370 \text{ paces} \times 0.8 \text{ m per pace} \\ &= 296 \text{ m}\end{aligned}$$

$$\begin{aligned}\text{Area of field} &= \text{side} \times \text{side} \quad (\text{square field}) \\ &= 296 \text{ m} \times 296 \text{ m} \\ &= 87\,616 \text{ m}^2\end{aligned}$$

You may need to review the area formula for a number of different plane shapes before you complete some of the problems in this topic.

Example 2

Nick knows that he covers 26 m with every 30 paces. He wants to compare the areas of two playgrounds that are in the shape of right angled triangle. He made the following sketches of the triangles:



- What is Nick's pace length?
- What are the dimensions of each playground?
- What are the areas of the two playgrounds?



Solutions:

a. Nick's pace length = $\frac{26}{30} = 0.866$

b. Figure A: height: $0.866 \times 40 = 34.66$; base: $0.866 \times 63 = 54.558$

Figure B: height: $0.866 \times 32 = 27.712$; base: $0.866 \times 84 = 72.744$

c. Area A = $\frac{1}{2} (54.588)(34.66) = 946 \text{ m}^2$

Area B = $\frac{1}{2} (72.744)(27.712) = 1007.9 \text{ m}^2$

Calculating Areas of Complex Shapes

One of the reasons for conducting a survey of a property is to determine the area of land included within the boundaries.

Land areas are calculated using one of the following metric units:

1 sq. metre	= 1 m x 1 m	= 1 m ²
1 hectare (ha)	= 100 m x 100 m	= 10 000 m ²
1 sq. kilometre	= 1000 m x 1000 m	= 1 000 000 000 m ²

The unit that is used depends on the size of the property being measured. Suburban building blocks of land, for instance are measured in square metres.

There are a number of ways of finding area, depending on whether it is calculated from a map or measurements obtained in a survey (**field measurements**). We will now look at calculations that are commonly necessary in elementary surveying work.

Field Measurements

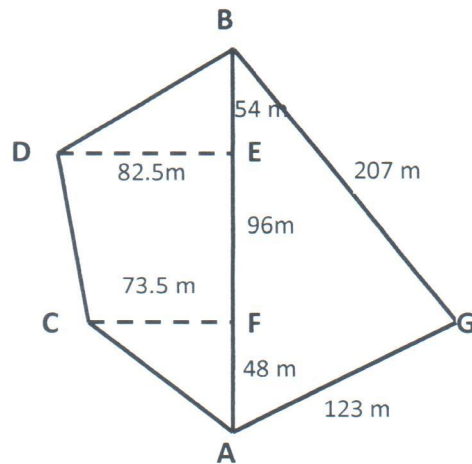
When a large **straight-edge** must be calculated, it is normal to **divide it into a number of smaller regions**. The areas of the smaller regions are then calculated and added to give the area of the entire region.



Using area of triangles and trapeziums

Example

A sketch of a farm is made and some field measurements are taken. Calculate the area of the farm.



Solution:

The farm can be conveniently divided into three triangles and a trapezoid.

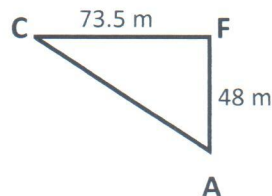
$$\text{Area for } \triangle AFC = \frac{1}{2} ab$$

$$A = \frac{1}{2} (AF)(CF)$$

$$= \frac{1}{2} (48 \text{ m})(73.5 \text{ m})$$

$$= \frac{1}{2} (3525) \text{ m}^2$$

$$= 1762.5 \text{ m}^2$$



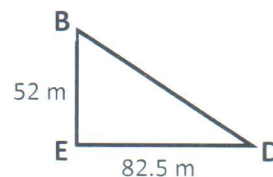
$$\text{Area for } \triangle DEB = \frac{1}{2} ab$$

$$A = \frac{1}{2} (EB)(DE)$$

$$= \frac{1}{2} (54 \text{ m})(82.5 \text{ m})$$

$$= \frac{1}{2} (4455) \text{ m}^2$$

$$= 2227.5 \text{ m}^2$$



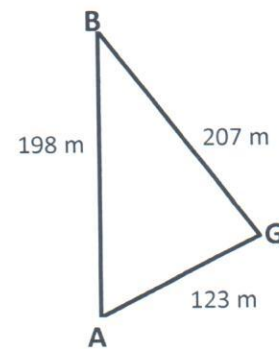


To find the area of $\triangle AGB$ we need to use Heron's formula.

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

First we find s using the formula: $s = \frac{a+b+c}{2}$

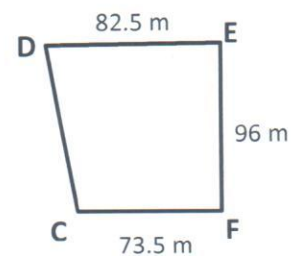
$$s = \frac{123+207+198}{2} = 264 \text{ m}$$



$$\begin{aligned} \text{So the area of } \triangle AGB &= \sqrt{264(264-207)(264-123)(264-198)} \\ &= \sqrt{264(57)(141)(66)} \\ &= \sqrt{140036683} \\ &= 11833.7 \text{ m}^2 \end{aligned}$$

To find the area of $CDEF$, use the formula: $A = \frac{a+b}{2}(h)$

$$\begin{aligned} \text{So the Area of } CFED &= \frac{82.5+73.5}{2}(96) \\ &= \frac{156}{2}(96) \\ &= 78(96) \\ &= 7488 \text{ m}^2 \end{aligned}$$

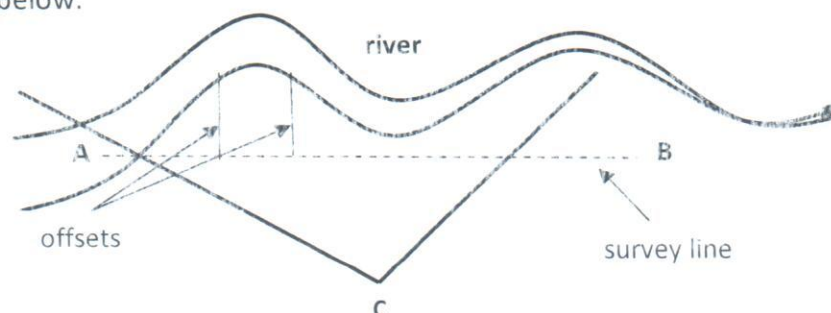


Now we can find the area of the farm by adding all the areas found.

$$\begin{aligned} \text{Area of the Farm} &= \triangle AFC + \triangle DEB + \triangle AGB + CFED \\ &= 1764 + 2227.5 + 11\,833.7 + 7488 \\ &= 23\,312.2 \text{ m}^2 \end{aligned}$$

In hectares, the area = **2.331 ha**

In reality, some regions that need to be measured have irregular or curved boundaries. An example is shown below.





In the field measurements, such boundaries are sketched by using a **survey line** (AB) and a number of equally spaced **offsets** from AB to the irregular boundary. In this case, the area of triangle ABC can be readily found.

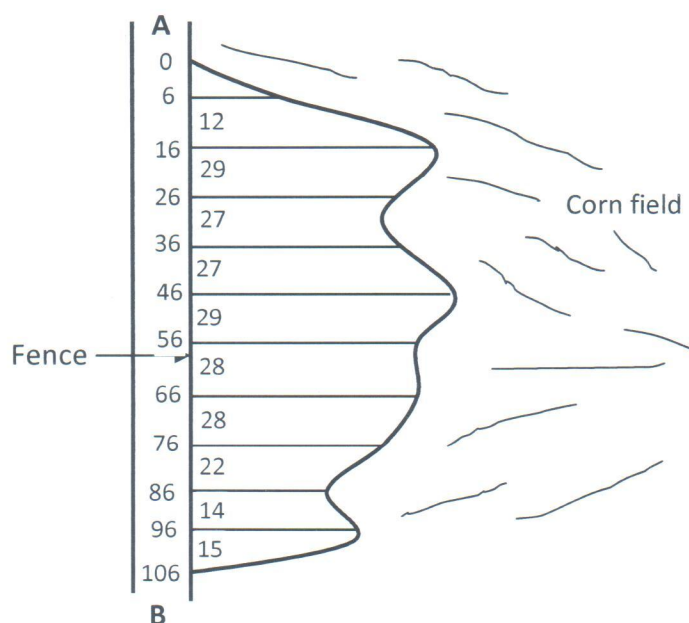
Using the mid-ordinate rule

The area of an irregular portion could be calculated by treating individually each of the regions by the offsets. However this will be time consuming. The formula below is used to calculate the area.

$$\text{Area of region} = \text{length of survey line} \times \text{Average of mid-ordinate lengths}$$

Example

Here are the field notes for the survey of a region between a fence and a line of a corn field. Use the information to find the area of the region. (Lengths are in metres.)



Solution:

Distance between mid-ordinates = 10 m

Length of survey line = 106 m

$$\begin{aligned}\text{Average length of mid-ordinates} &= \frac{12 + 29 + 27 + 27 + 29 + 28 + 28 + 22 + 14 + 15}{10} \\ &= \frac{231}{10} \\ &= 23.1 \text{ m}\end{aligned}$$

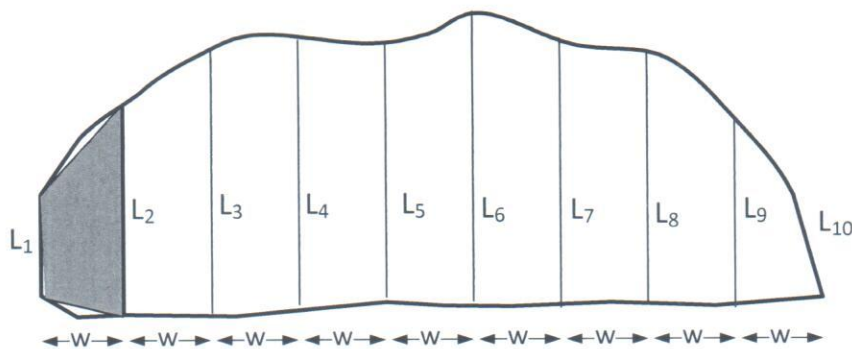


$$\begin{aligned}
 \text{Area of region} &= \text{length of survey line} \times \text{Average of mid-ordinate lengths} \\
 &= (106 \text{ m})(23.1 \text{ m}) \\
 &= 2448.6 \text{ m}^2
 \end{aligned}$$

Using Trapezoidal rule

This method is also used by surveyors to calculate irregular areas. It is a variation of the mid-ordinate rule, but uses measurements at the edges of the offset strips.

An irregular area is divided into strips of equal width (w) and the edges of the strips are noted as $L_1, L_2, L_3, \dots, L_{10}$ as shown in the diagram.



While applying the trapezoidal rule, boundaries between the ends of the ordinates are assumed to be straight. Thus the areas enclosed between the base line and the irregular boundary lines are considered as trapezoids.

The first trapezoid (shaded strip) has an area = $\frac{1}{2}(L_1 + L_2)(w)$

The second Trapezoid has an area = $\frac{1}{2}(L_2 + L_3)(w)$, the third has an area = $\frac{1}{2}(L_3 + L_4)(w)$ and

so on to the last trapezoid having its area = $\frac{1}{2}(L_9 + L_{10})(w)$

Adding all these nine areas will give us the area of the region.

$$\begin{aligned}
 \text{Area of the region} &= \frac{1}{2}[(L_1 + L_2)(w) + \frac{1}{2}(L_2 + L_3)(w) + \frac{1}{2}(L_3 + L_4)(w) \dots + \frac{1}{2}(L_9 + L_{10})(w)] \\
 &= \frac{1}{2}(w)(L_1 + L_2 + L_2 + L_3 + L_3 + L_4 + L_4 + L_5 + L_5 + L_6 \dots + L_{10}) \\
 &= \frac{1}{2}w(L_1 + L_{10} + 2(L_2 + L_3 + L_4 + L_5 + L_6 + L_7 + L_8 + L_9 + L_{10}))
 \end{aligned}$$



Thus, the approximate area of an irregular region, if divided into equal strips is calculated by using the formula:

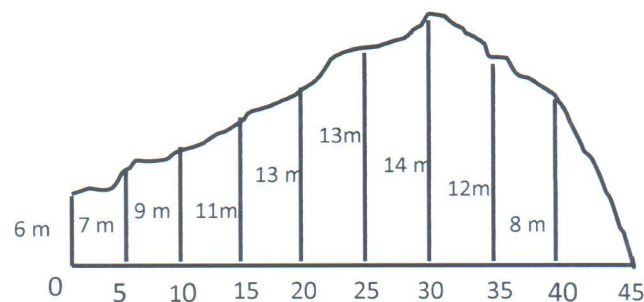
$$\text{Area} = \frac{\text{width}}{2} \times (\text{End lengths} + 2 \times \text{Total of mid lengths})$$

L_1 and L_{10} are end lengths and $L_2, L_3, L_4 \dots$ and L_{10} are middle lengths.

This formula refers to the trapezoidal Rule.

Example

Find the area of this region.



Solution:

The last length in this case, after the 8 m is 0 m. The width of each strip is 5 m.

Sum of end lengths = $6 + 0 = 6$

Total of middle lengths = $7 + 9 + 11 + 13 + 13 + 14 + 12 + 8 = 87$

To find the area of the region $\text{Area} = \frac{\text{width}}{2} (\text{sum of end lengths} + 2(\text{sum of middle lengths}))$

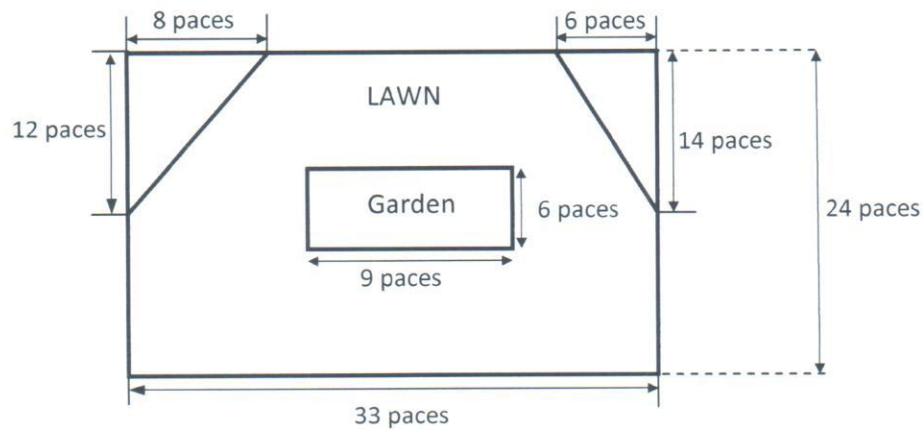
$$\begin{aligned} \text{Area} &= \frac{5}{2} [6 + 2(87)] \\ &= \frac{5}{2} (6 + 174) \\ &= \frac{5}{2} (180) \\ &= 5(90) \\ &= 450 \text{ m}^2 \end{aligned}$$

Now do the learning activity.

**Learning Activity 12.1.4.1**

20 minutes

1. Kathy decides to position three garden plots at her house. She used the garden hose to mark the edges of the gardens and drew the following sketch as a result.

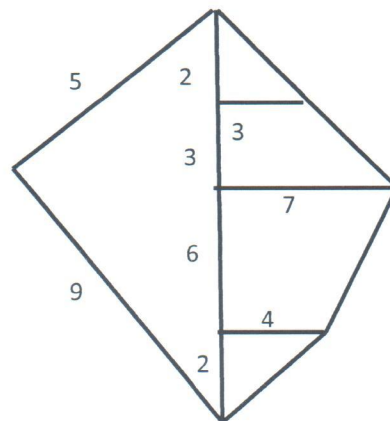


Kathy takes 39 paces to cover 25 metres.

- What is Kathy's pace length?
- What are the dimensions of her backyard?
- What is the area of the backyard?
- What are the dimensions of the corner garden beds?
- What is the area of each corner garden bed?
- What is the area of the centre garden bed?
- What is the area of the lawn?



2. Calculate the area of the flower garden below. Measurements are in metres.



3. Estimate the area of this proposed kid's play area with 8 m widths.

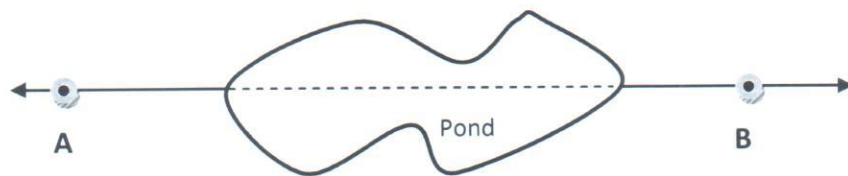




12.1.4.2: Surveying Around an Obstacle

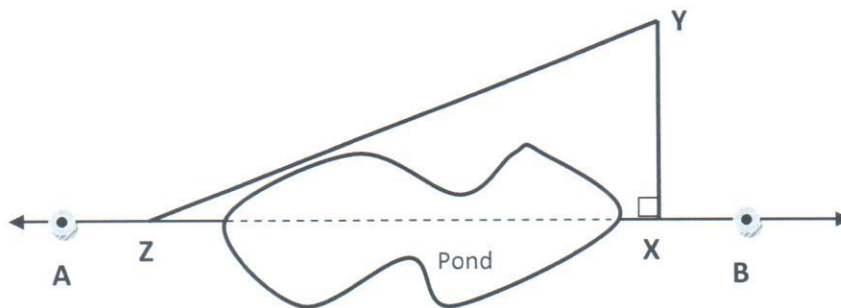
There are sometimes unavoidable obstacles or obstructions met when measuring or ranging a survey line. Some obstacles can be measured around, while others cannot. Triangulation and offsets are useful tools in landscape design, especially for irregularly shaped yards. Both help you plot the exact position of boundaries and features, which can then be added to a scale plan.

We will begin by examining those that can be measured around, such as the pond obstructing the survey line AB below.



The Offset Method

One way to measure around the pond is to make an **offset**, XY near the edge of the pond. XZ is then measured back to the survey line, meeting it at Z. This forms the right triangle XYZ.



We need to know the distance ZX, which can be found using Pythagoras Theorem.

Example

Use the diagram above. Find the distance **ZX**, if **XY** and **YZ** are 18.4 m and 65.2 m respectively.

Solution:

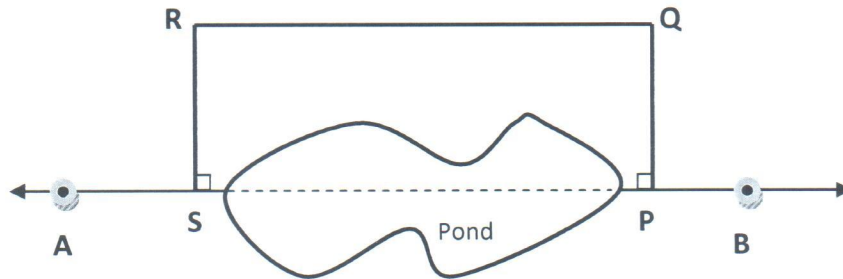
Given $XY = 18.4 \text{ m}$ and $YZ = 65.2 \text{ m}$

$$\begin{aligned}\text{In } \triangle XYZ \quad YZ^2 &= ZX^2 + XY^2 \\ 65.2^2 &= ZX^2 + 18.4^2 \\ ZX^2 &= 65.2^2 - 18.4^2 \\ ZX^2 &= 3912.48 \\ ZX &= 62.5 \text{ m}\end{aligned}$$



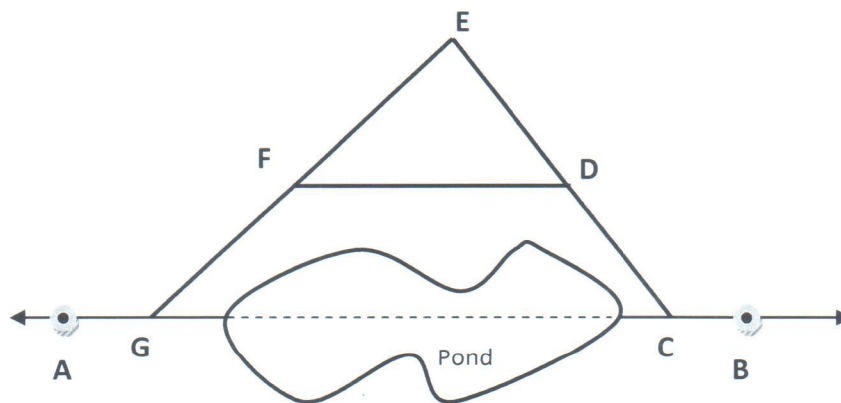
Below are other ways on how to use offsets to find the distance around obstacles.

- a) After the offsets PQ and SR are ranged, RQ can be measured. RQ is the required distance, SP.



- b) It is possible to use similar triangles to calculate the required distance.

This is done in the following way. Point C is set at a convenient place on the survey line. Next, a point E is set at some distance clear of the obstruction. Point G is then set on the survey line so that EG is clear of the obstruction. Distances GE and CE are carefully measured and bisected at points F and D respectively.



This means that $\triangle FED$ and $\triangle GEC$ are similar. In similar triangles, the ratio of corresponding sides is equal. $\frac{FE}{GE} = \frac{ED}{EC} = \frac{FD}{GC} = \frac{1}{2}$. So $GC = 2 FD$ (the required distance FD which can easily be measured.)

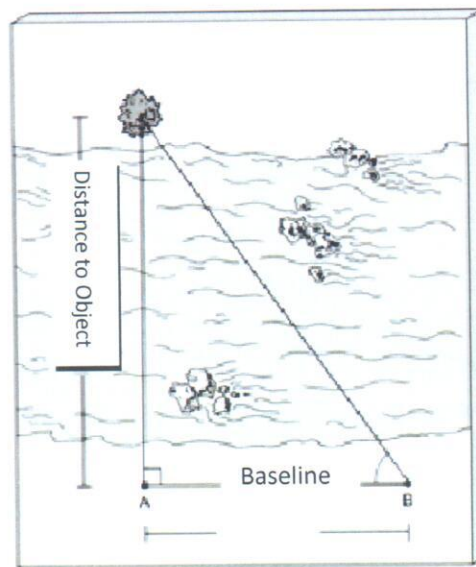
This procedure is also used by surveyors as a method of setting out parallel lines.

The Triangulation Method

Triangulation is a method of surveying large areas of land using triangles.



Imagine trying to measure the distance to a tree on the other side of a river.



The most direct method is to lay a tape across the river, but that's not the simplest way. A smart surveyor would make the measurement by visualizing an *imaginary* triangle (hence *triangulation*), sighting the tree on the far side of the river from two positions on the near side.

The simplest possible triangle is a right triangle, in which one of the angles is exactly 90° , so it is usually convenient to set up one observation position directly opposite the object, as at point A. The surveyor then moves to another observation position at point B, noting the distance covered between points A and B. This distance is called the **baseline** of the imaginary triangle. Finally, the surveyor, standing at point B, sights toward the tree and notes the angle at point B between this sightline and the baseline. No further observations are required. Knowing the value of one side (AB) and two angles (the right angle itself, at point A, and the angle at point B) of the right triangle, the surveyor can geometrically construct the remaining sides and angles and so establish the distance from A to the tree.

Surveyors often use simple geometry and trigonometry to estimate the distance to a faraway object.

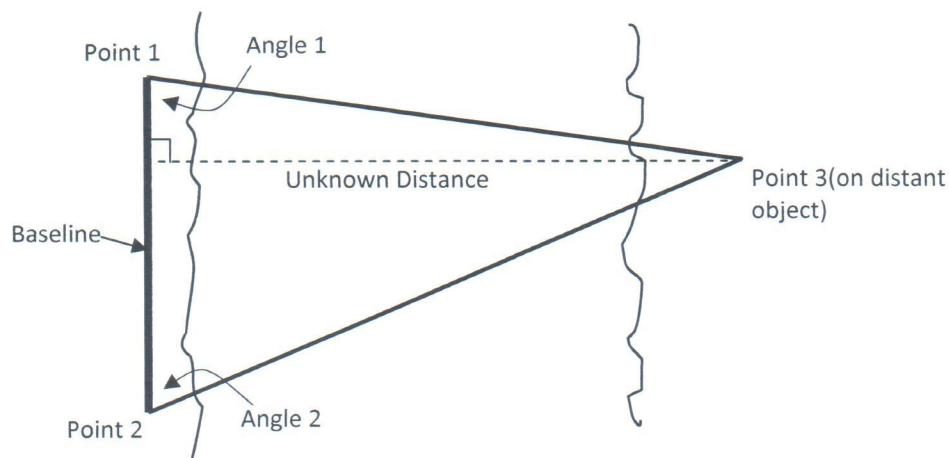
How to Measure Distances Using Triangulation

Example

Measuring the distance across a river

Step 1: Create a baseline and measure its length.

Step 2: Measure the angles from the end of the baseline (angles 1 and 2).



Step 3: Make a scale drawing of the triangle.

Use an appropriate scale. Let us say our baseline in the above diagram is 15 m and the angles measured are 82° for angle 1 and 67° for angle 2.

Scale 1 cm = 5 m

Step 4: In the scale drawing, measure the unknown distance in cm using a ruler.

In our scale drawing the unknown distance is 5.2 cm

Step 5: Use proportions to find the unknown distance.

Make a proportion of our scale to the unknown distances in the two diagrams.

$$\frac{5.2}{3} = \frac{x}{15}$$

$$3x = 5.2(15) \quad \text{Cross multiply}$$

$$3x = 78 \quad \text{Divide both sides by 3}$$

$$x = 26$$

Therefore, the distance from the baseline to the point across the river is approximately 26 m.

Now do the learning activity on the next page.

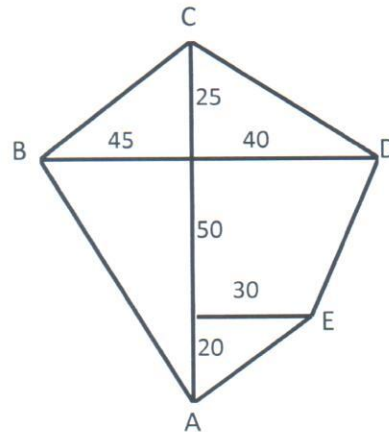


Learning Activity 12.1.4.2

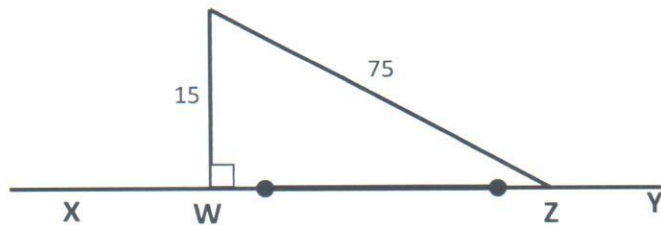


20 minutes

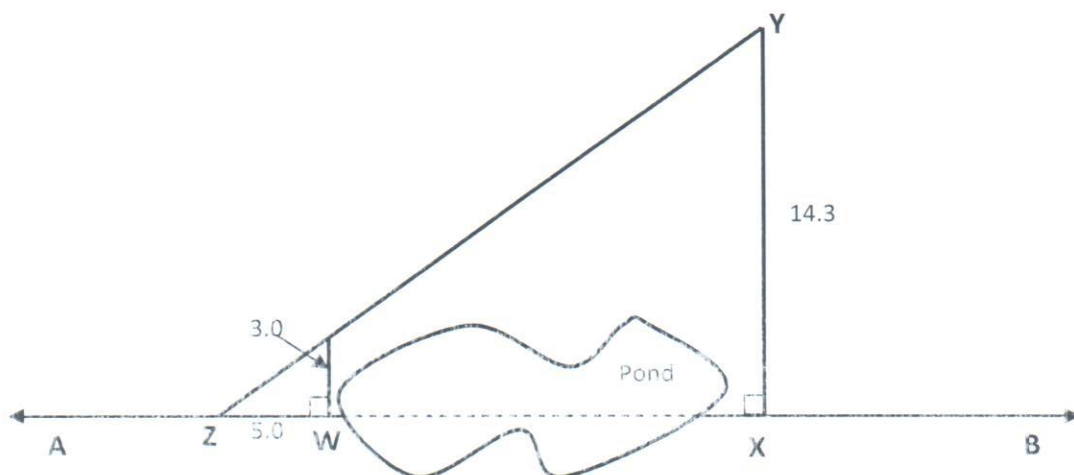
1. Find the area of the farm below. All measurements are in metres.



2. A small drainage flows across a survey line XY. Find the distance WZ.



3. A pond obstructs the survey line AB. Calculate the length of WX.





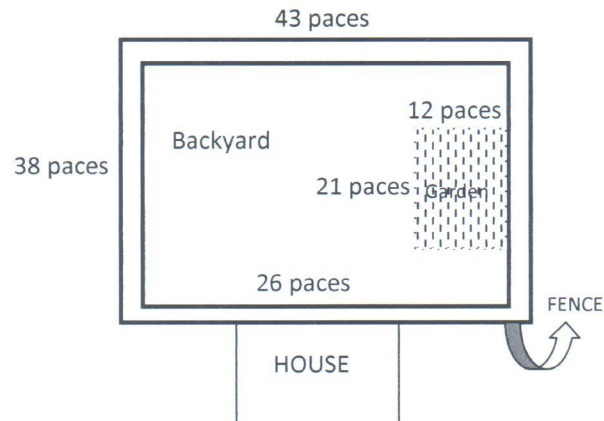
SUMMATIVE TASK 12.1.4



20 minutes

Solve the following problems.

1. Marie calculates that she takes on average, 28 paces to cover 20 m. She decides to help her parents measure their back yard and the garden bed it contains. She makes some notes as shown here.

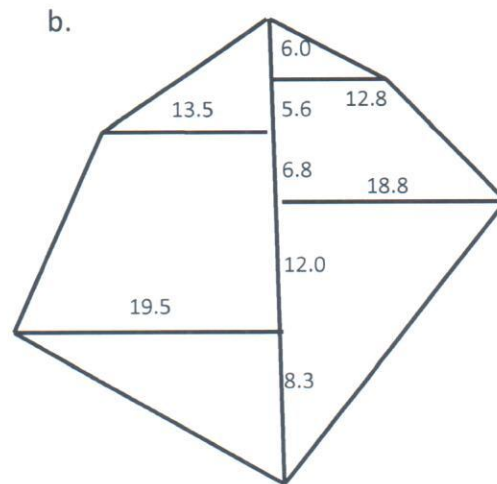
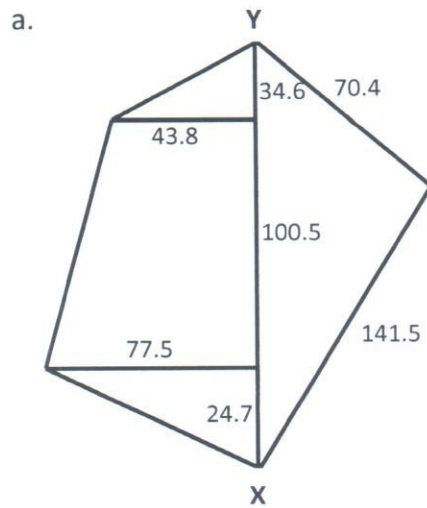


Use the sketch to calculate:

- a. the dimensions of the backyard.
- b. the dimensions of the garden.
- c. the area of the back yard.
- d. the area of the garden
- e. the length of the fence.

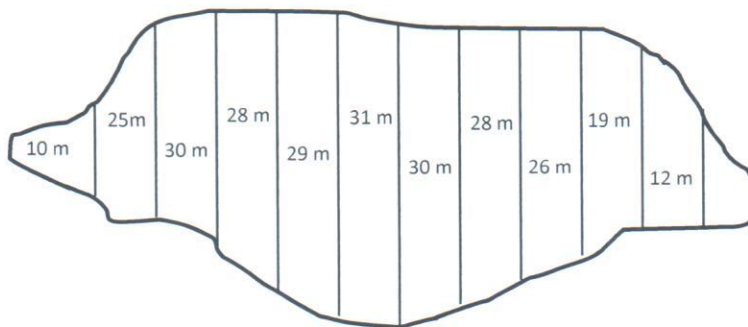


2. Calculate the area of the field notes.



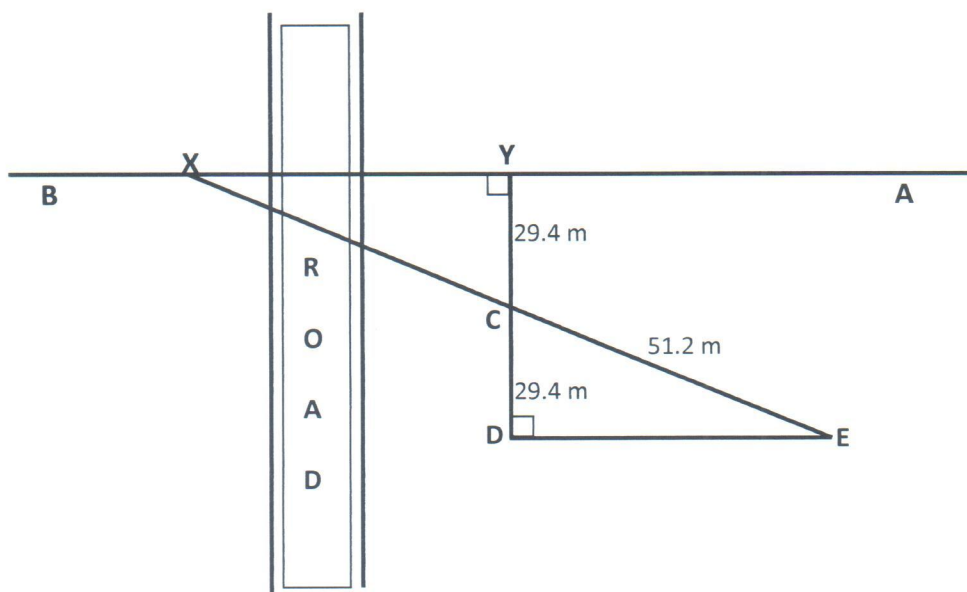
3. Find the area using the trapezoidal rule.

5m wide strips

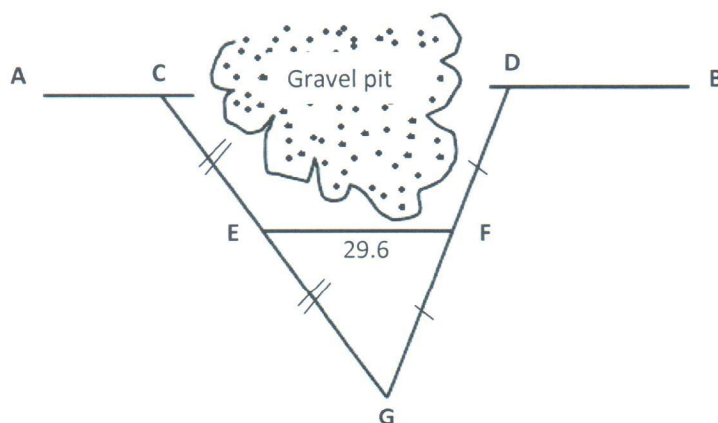




4. A small road lies across a survey line AB. Calculate the missing distance XY.



5. A gravel pit lies across the survey line AB. Work out the missing length CD.





UNIT SUMMARY

This summary outlines the important key ideas and concepts to be remembered.

Similar Figures

- Similar figures have all pairs of corresponding angles equal and corresponding sides in equal ratio.
- To show that triangles are similar, we show either that all corresponding angles are equal or that all parts of corresponding sides are in equal ratio.
- For other figures, it is necessary to show that both properties are true.

Scale Factors

- The scale factor allows us to solve problems using similar figures.
- Heights of objects, such as trees, that are not easily measured can be determined by constructing similar triangles.

Perimeter and Area of Plane Shapes

- The perimeter of a plane shape is the total length of the boundary forming the figure.
- The perimeter of a shape is the distance around its edges – the distance you would have to travel to walk once around the shape along its edge. In general, the perimeter of a shape is the sum of the lengths of its sides.
- To find the perimeter of some irregular shapes you will need to calculate a side length using a known formula or Pythagoras Theorem.
- The perimeter of an object in a plane is the length of its boundary.
- The area of an object is the amount of surface that the object occupies.
- The area of a shape is the total number of square units that fill the shape.

Area formulas that you will need to remember

- Square $A = s^2$
- Rectangle $A = l \times b$
- Triangle $A = \frac{1}{2}bh$
- Parallelogram $A = bh$
- Rhombus $A = \frac{1}{2} \times D \times d$
- Trapezium $A = \frac{1}{2} \times (a + b) \times h$

Composite areas are calculated by breaking the plane shape into smaller figures.

Surface Area and Volume of Solids

- Surface area measures the area of a surface – essentially, it is the same as area. The unit of measure for surface area is the square unit, just as in area.

Volume

- The volume is the amount of space inside a solid.
- Volume for three-dimensional figures is the equivalent of area for two-dimensional figures.



- Volume is a three-dimensional measurement, that is, it measures the combined length, width, and height of a figure. Because planes, polygons, and other two-dimensional figures do not have height, they have no volume. Only solids and figures with dimension greater than or equal to three have volume.

Volume formulas that you need to remember.

- Cube $V = s^3$
- Rectangular Prism $V = l \times b \times h$
- Sphere $V = \frac{4}{3} \pi r^3$
- Cone $V = \frac{1}{3} \pi r^2 h$
- Cylinder $V = \pi r^2 h$

Any other prism has its volume calculated by using the formula $V = A \times h$, where A is the area of the base and h is the height.

Surveying

- Land surveying involves measuring, sketching and recording of lengths and areas, and showing information using suitable scale on a map or a plan.
- Pace is a length of one large step and is one of the earliest forms of measuring length.
- Offset surveys are used to measure irregular blocks of land.
- Base line is the length of a line joining the two end points of an irregular region being measured.
- Offset distance is the measurement of the perpendicular distance to the base line.
- The Mid-ordinate rule formula:

Area of region = length of survey line x Average of mid – ordinate lengths

- The Trapezoidal rule states that the approximate area of an irregular region if divided into equal strips is calculated using the formula:

$$\text{Area} = \frac{\text{width}}{2} \times (\text{End lengths} + 2 \times \text{Total of mid lengths})$$

REVISE SECTIONS 12.1.1 TO 12.1.4 THEN DO MODULE ASSESSMENT 1

**ANSWERS TO LEARNING ACTIVITIES AND SUMMATIVE TASKS****Learning Activity 12.1.1.1**

1. a. 20 b. 11.5 c. 32
2. $\frac{1}{4}$
3. a. 1:2 b. 6.5 c. 5
4. a. 6 b. 42 c. 7 d. 21

Learning Activity 12.1.1.2

1. a. $\frac{2}{3}$ b. $\frac{20}{3}$ c. $\frac{3}{2}$ d. $\frac{9}{2}$
2. a. $\frac{1}{2}$ b. $\frac{13}{2}$ or 6.5 c. 5
3. 5:3
4. a. 4:5 b. 16:25
5. 8 cm

Summative Task 1

1. a. 3:2
b. 28
c. 42
d. 40
e. 90
f. Scale factor of side = 1.5 Scale factor of area = 2.25
The scale factor of the sides is increased by 1.5 whilst the scale factor of the area is increased by 2.25.
2. a. SF = 1:2 or 0.5; x = 4.5 b. SF = 4; x = 33.6
c. SF = 1:7; x = 1 d. SF = 1:4; x = 4
e. SF = 2 ; x = 8 f. SF = $\frac{4}{3}$; x = 4.5



-
3. a. 1.5 b. 0.6
c. Area = 8.75; Perimeter = 15
d. Area = 19.69; Perimeter = 22.5
e. Perimeter is enlarged by 1.5.
f. Area is enlarged by 2.25.
4. a. 3
b. 1: 3
c. Area = 28 Perimeter = 22
d. Area = 252; Perimeter = 66
e. Perimeter is enlarge by 3
f. Area is enlarged by 9.
5. 100
6. 15
7. 80
-

Learning Activity 12.1.2.1

1. a. 6.4 b. 5.7 c. 6.6
2. 8
3. 9.43
4. NOT a right triangle
5. 1963.5
-

Learning Activity 12.1.2.2

1. 107.734
2. 138.56
3. 102.3
4. 226 909 ft²
5. 23.02 cm²
-

**Learning Activity 12.1.2.3**

1. a. 27.5 b. 156.89
 2. $\sqrt{8}$
 3. 65 cm^2
 4. 172.59 m^2
 5. 2449.5 cm^2
-

Learning Activity 12.1.2.4

1. a. 34 b. 10
 2. 15.7
 3. 695.52
 4. Width = 26; Length = 29
 5. 14 or 18 cm^2
-

Summative Task 2

1. a. 21.85 b. 27.58 c. 10.4 d. 3.4
 2. a. $40\sqrt{3}$ b. $150\sqrt{3}$
 3. Perimeter = 112 mm
 4. Area = 23.95 m^2
 5. Area = 1340.81
 6. Approximately 27.7 or 28 gallons
 7. Area = 10.9 or 11 ft^2
 8. K 228
-

Learning Activity 12.1.3.1

1. a. 168 b. 54 c. 282.6 d. 273.18
 2. 1256.00
 3. 311.40 cm
 4. a. 93.73 cm^2 b. 124.16 cm^2
 5. 339.12 or 340 cm
-

**Learning Activity 12.1.3.2**

1. 5376
 2. a. 120 b. 4710
 3. 615.44
 4. 40
 5. 3.085 or 3 cm
 6. 33.33 m^3
 7. $142,346.67 \text{ cm}^3$
-

Summative Task 3

1. a. Perimeter = 60 cm ; Area = 150 cm^2
b. Perimeter = 14 cm; Area = 48 cm^2
 2. a. Area = 50.24 cm^2
b. Area = 28.26 cm^2
 3. a. Volume = 3 cm^3 ; Surface Area = 14 cm^2
b. Volume = 301.6; Surface Area = 251.20
 4. Surface area = 96 cm^2
 5. a. 180 cm^3 b. 6154.4 cm^3
 6. 1256 cm^2
 7. 10.64 m
 8. 720 m^3
 9. Lateral area = 1045.18 cm^2 ; Surface area = 1811.34 cm^2 ; Volume = 5714.80 cm^3
-

Learning Activity 12.1.4.1

1. a. 0.64 m b. 21.1 m by 15.4 m c. 324 to 326 m^2
d. Garden A-base = 5.1; height = 7.7;
Garden B - base = 3.8 or 4; height = 8.96 or 9
e. A - 19.66 m^2 ; B = 17.20 m^2
f. 22.1
g. 265.42
 2. 16.07
 3. 616
-

**Learning Activity 12.1.4.2**

1. 4687.5 m^2 2. 73.5 3. 18.8
-

Summative Task 4

1. a. 30.53 by 27.14 b. 15 by 8.5 c. 823.70 m^2
d. 127.03 m^2 e. 115.02 m
2. a. 12789 m^2 b. 895 m^2
3. 1345 m^2
4. 41.92 m
5. 59.2 m
-

END OF UNIT MODULE



References

- Grove, Margaret (2010), *Mathematics in Focus (Preliminary Course)*, Cengage Learning Australia, Melbourne
- Rowland Robert, (2013) *Maths Quest Preliminary (Mathematics General)*, John Wiley and Sons Australia, Queensland
- Jonnes S. B. and Couchman K.E (2006) *2 Unit Mathematics Book 1*, Pearson Education Australia, Melbourne
- Badie, Ross and Swift Stephen (1994) *12a QMaths*, Moreton Bay Publishing, Melbourne
- Mc Rae Christine, Esther Gawac Ehava et al. (2013) *Save Buk 12 General Mathematics (PNG Upper Secondary)*, Oxford University Press, Melbourne

On-line Resources

- <http://www.cliffsnotes.com/math/geometry/similarity/similar-triangles-perimeters-and-areas>
- <http://lifeng.lamost.org/courses/astrotoday/CHAISSON/AT301/HTML/AT30105.HTM>
- <http://chatt.hdsb.ca/~komljenovicm/snc%201d%20triangulation.htm>
- <http://mathforum.org/paths/measurement/meastriangm.html>
- http://www.mathsteacher.com.au/year10/ch06_geometry/05_similar/figures.htm#scalefact
or
http://www.mathsteacher.com.au/year10/ch06_geometry/05_similar/figures.htm#scalefact
or
<http://www.mathwarehouse.com/geometry/similar/triangles/sides-and-angles-of-similar-triangles.php>
- <http://www.cimt.plymouth.ac.uk/projects/mepres/book8/book8int.htm>
- SparkNotes Editors. "SparkNote on Geometry: Measurements." SparkNotes LLC. n.d..
- <http://www.sparknotes.com/math/geometry2/measurements/> (accessed June 11, 2015).
- http://www.mathsteacher.com.au/year10/ch14_measurement/15_csa_cylinder/20csa.htm
- <http://www.mathopolis.com/questions/q.php?id=259>
- <http://www.ask-math.com/herons-formula.html>
- <http://www.nios.ac.in/media/documents/secmathcour/eng/chapter-21.pdf>

FODE PROVINCIAL CENTRES CONTACTS

FODE PROVINCIAL CENTRES

PC N O.	FODE PROVINCIAL CENTRE	ADDRESS	CONTACTS/ CUG PHONES	WIRELESS PHONES	SENIOR CLERK	CUG PHONE
1	DARU	P. O. Box 68, Daru	72228146	77522841	Mr Kevin Sere	72229047
2	KEREMA	P. O. Box 86, Kerema	72228124	77522842	Mr David Saria	72229049
3	CENTRAL	C/- FODE HQ	72228110	77522843	Mr Aubi Elodo	72229050
4	ALOTAU	P. O. Box 822, Alotau	72228130	77522844	Mr Albi Bapera	72229051
5	POPONDETTA	P. O. Box 71, Popondetta	72228138	77522845	Mr Stansen Sevese	72229052
6	MENDI	P. O. Box 237, Mendi	72228142	77522846	Mr Wari Tange	72229053
7	GOROKA	P. O. Box 990, Goroka	72228116	77522847	Ms Ovin Tuki	72229054
8	KUNDIAWA	P. O. Box 95, Kundiawa	72228144	77522848	Mr Denge Gundu	72229056
9	MT HAGEN	P. O. Box 418, Mt. Hagen	72228148	77522849	Mr Robert Maki	72229057
10	VANIMO	P. O. Box 38, Vanimo	72228140	77522850	Mrs Isabella Danti	72229060
11	WEWAK	P. O. Box 583, Wewak	72228122	77522851	Mr David Wombui	72229062
12	MADANG	P. O. Box 2071, Madang	72228126	77522852	Mrs Applonia Bogg	72229063
13	LAE	P. O. Box 4969, Lae	72228132	77522853	Ms Cathrine Kila	72229064
14	KIMBE	P. O. Box 328, Kimbe	72228150	77522854	Mrs Bernadette Litom	72229065
15	RABAU	P. O. Box 83, Kokopo	72228118	77522855	Mrs Verlyn Vavai	72229067
16	KAVIENG	P. O. Box 284, Kavieng	72228136	77522856	Mr John Lasisi	72229069
17	BUKA	P. O. Box 154, Buka	72228108	77522857	Mr Marlyne Meiskamel	72229073
18	MANUS	P. O. Box 41, Lorengau	72228128	77522858	Ms Roslyn Keket	72229080
19	NCD	C/- FODE HQ	72228134	77522859	Mrs Marina Tomiyavau	72229081
20	WABAG	P. O. Box 259, Wabag	72228120	77522860	Mr Salas Kamberan	72229082
21	HELA	P. O. Box 63, Tari	72228141	77522861	Mr Ogai John	72229083
22	JIWAKA	c/- FODE Hagen	72228143	77522862	Joseph Walep	72229085

SUBJECT AND GRADE TO STUDY

Grade Levels	Subjects
Grades 7 and 8	1. English
	2. Mathematics
	3. Science
	4. Social Science
	5. Making a Living
	6. Personal Development
	7. English
Grades 9 and 10	1. English
	2. Formal Mathematics
	3. Practical Mathematics
	4. Science
	5. Social Science
	6. Commerce
	7. Design and Technology- Computing
	8. Personal Development
Grades 11 and 12	1. English
	• Applied English
	• Language and Literature
	2. Mathematics
	• Mathematics A
	• Mathematics B
	3. Science
	4. Social Science
	• History
	• Geography
	• Economics
	5. Business Studies
	6. Personal Development
	7. ICT

REMEMBER:

- For Grades 7 and 8, you are required to do all six (6) courses.
- For Grades 9 and 10, you must study English, Mathematics, Science, Personal Development, Social Science and Commerce. Design and Technology-Computing is optional.
- For Grades 11 and 12, you are required to complete seven (7) out of thirteen (13) courses to be certified. Your Provincial Coordinator or Supervisor will give you more information regarding each subject.

Certificate in Matriculation

CORE COURSES

Basic English
English 1
English 2
Basic Maths
Maths 1

Maths 2

History of Science & Technology

OPTIONAL COURSES

Science Streams: Biology

Chemistry, Physics and Social Science Streams:

Geography, Introduction to Economics and Asia and the Modern World

REMEMBER:

You must successfully complete 8 courses; 5 compulsory and 3 optional